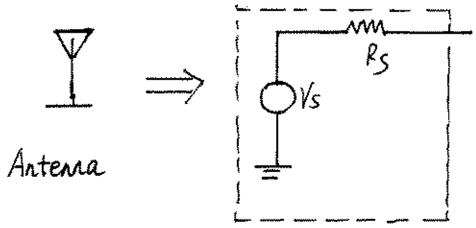
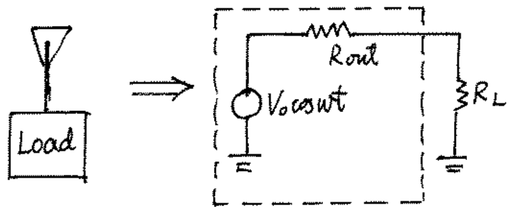
1) 

Thevenin Equivalent:

$V_s = V_0 \cos \omega t$

$R_s = R_{out}$



Average power delivered to load $= (I_{RMS})^2 R_L$,

$I_{RMS} = \frac{V_{RMS}}{R_{out}+R_L}$, $V_{RMS} = \frac{V_0}{\sqrt{2}}$ $\Rightarrow$ $I_{RMS} = \frac{V_0}{\sqrt{2}(R_{out}+R_L)}$

Average power $= (I_{RMS})^2 R_L = \frac{V_0^2 R_L}{2(R_{out}+R_L)^2}$ (Eq. 1)

Plot of Average Power

When R_L is small, Eq. 1 is small.

When R_L is large, Eq. 1 is also small.

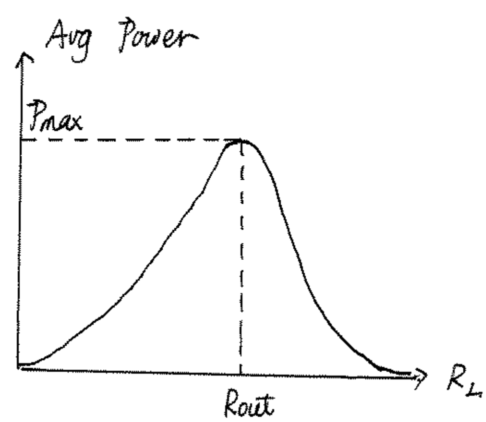
So for some R_L between zero and infinity, the average power will reach its peak. Lets take the derivative of Eq. 1 with respect to R_L to find the optimum R_L.

$$\frac{\partial}{\partial R_L}\left[\frac{V_0^2 R_L}{2(R_{out}+R_L)^2}\right] = \frac{V_0^2}{2(R_{out}+R_L)^2} - \frac{V_0^2 R_L}{(R_{out}+R_L)^3}$$

Setting it to zero and solve for R_L

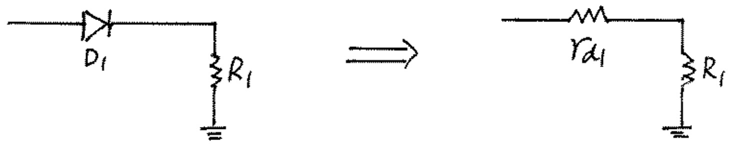
$$\frac{V_0^2}{2(R_{out}+R_L)^2} = \frac{V_0^2 R_L}{(R_{out}+R_L)^3} \Rightarrow \frac{(R_{out}+R_L)}{2} = R_L$$

$$\Rightarrow R_{out}+R_L = 2R_L \Rightarrow R_L = R_{out}$$



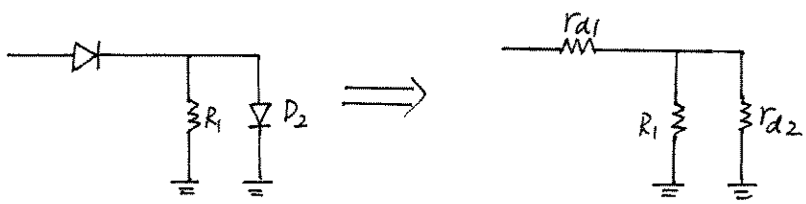
2) In small signal operation, a diode can be replaced by a linear resistor if changes are small.

a).



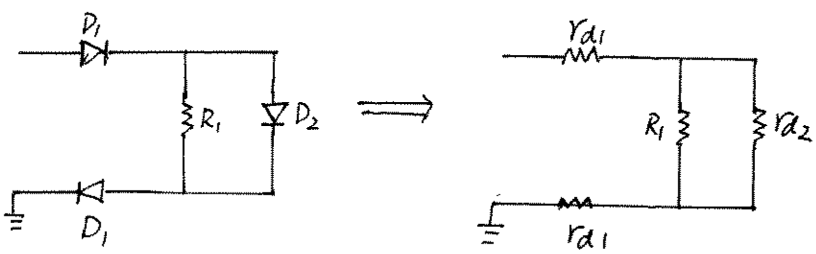
$R_{in} = r_{d1} + R_1$

b).



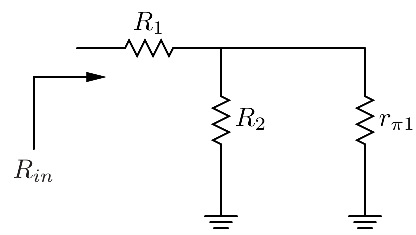
$R_{in} = r_{d1} + R_1 // r_{d2}$ (// means in parallel)

c).



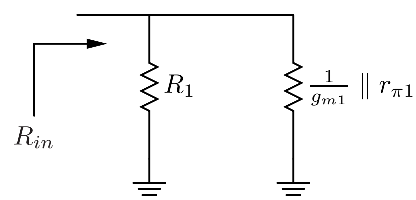
$R_{in} = 2r_{d1} + R_1 // r_{d2}$

5.3 (a) Looking into the base of Q_1 we see an equivalent resistance of $r_{\pi 1}$, so we can draw the following equivalent circuit for finding R_{in}:



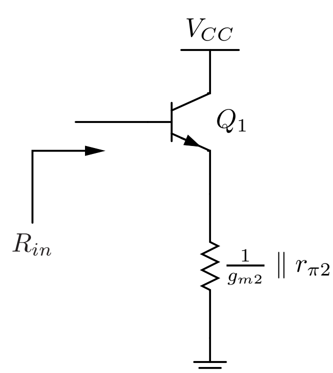
$$R_{in} = \boxed{R_1 + R_2 \parallel r_{\pi 1}}$$

(b) Looking into the emitter of Q_1 we see an equivalent resistance of $\frac{1}{g_{m1}} \parallel r_{\pi 1}$, so we can draw the following equivalent circuit for finding R_{in}:



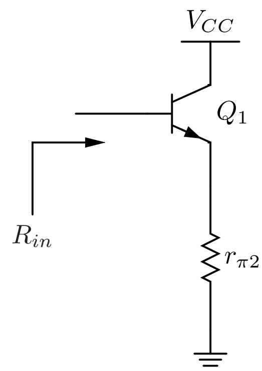
$$R_{in} = \boxed{R_1 \parallel \frac{1}{g_{m1}} \parallel r_{\pi 1}}$$

(c) Looking down from the emitter of Q_1 we see an equivalent resistance of $\frac{1}{g_{m2}} \parallel r_{\pi 2}$, so we can draw the following equivalent circuit for finding R_{in}:



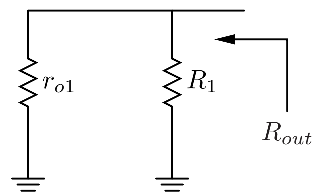
$$R_{in} = \boxed{r_{\pi 1} + (1 + \beta_1)\left(\frac{1}{g_{m2}} \parallel r_{\pi 2}\right)}$$

(d) Looking into the base of Q_2 we see an equivalent resistance of $r_{\pi 2}$, so we can draw the following equivalent circuit for finding R_{in}:



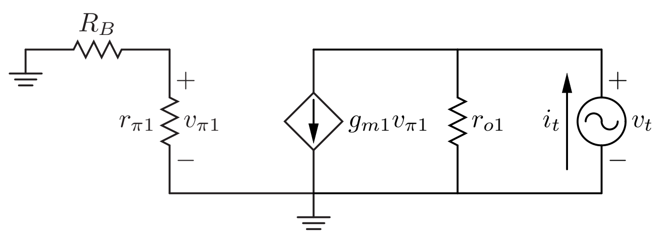
$$R_{in} = \boxed{r_{\pi 1} + (1+\beta_1) r_{\pi 2}}$$

5.4 (a) Looking into the collector of Q_1 we see an equivalent resistance of r_{o1}, so we can draw the following equivalent circuit for finding R_{out}:



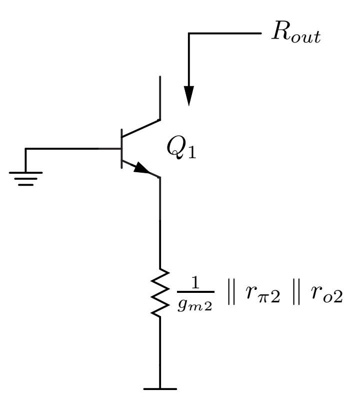
$$R_{out} = \boxed{r_{o1} \parallel R_1}$$

(b) Let's draw the small-signal model and apply a test source at the output.



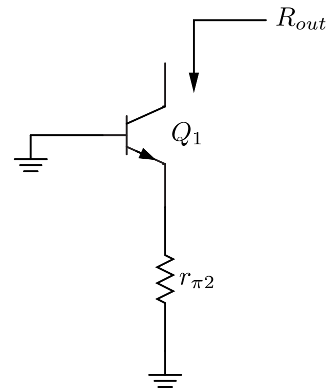
$$i_t = g_{m1}v_{\pi 1} + \frac{v_t}{r_{o1}}$$
$$v_{\pi 1} = 0$$
$$i_t = \frac{v_t}{r_{o1}}$$
$$R_{out} = \frac{v_t}{i_t} = \boxed{r_{o1}}$$

(c) Looking down from the emitter of Q_1 we see an equivalent resistance of $\frac{1}{g_{m2}} \parallel r_{\pi 2} \parallel r_{o2}$, so we can draw the following equivalent circuit for finding R_{out}:



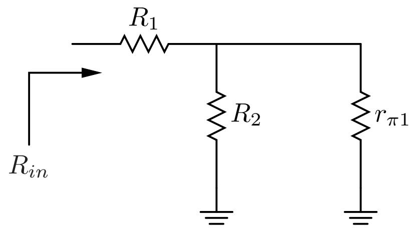
$$R_{out} = \boxed{r_{o1} + (1 + g_{m1}r_{o1})\left(r_{\pi 1} \parallel \frac{1}{g_{m2}} \parallel r_{\pi 2} \parallel r_{o2}\right)}$$

(d) Looking into the base of Q_2 we see an equivalent resistance of $r_{\pi 2}$, so we can draw the following equivalent circuit for finding R_{out}:



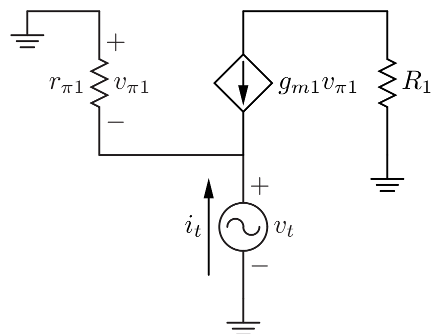
$$R_{out} = \boxed{r_{o1} + (1 + g_{m1} r_{o1})\,(r_{\pi 1} \parallel r_{\pi 2})}$$

5.5 (a) Looking into the base of Q_1 we see an equivalent resistance of $r_{\pi 1}$, so we can draw the following equivalent circuit for finding R_{in}:



$$R_{in} = \boxed{R_1 + R_2 \parallel r_{\pi 1}}$$

(b) Let's draw the small-signal model and apply a test source at the input.



$$i_t = -\frac{v_{\pi 1}}{r_{\pi 1}} - g_{m1} v_{\pi 1}$$

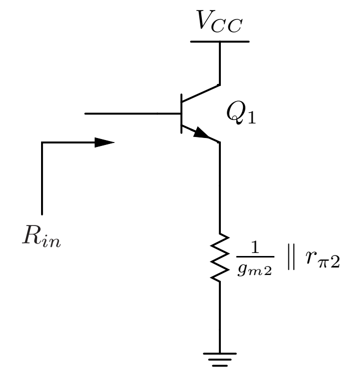
$$v_{\pi 1} = -v_t$$

$$i_t = \frac{v_t}{r_{\pi 1}} + g_{m1} v_t$$

$$i_t = v_t \left(g_{m1} + \frac{1}{r_{\pi 1}} \right)$$

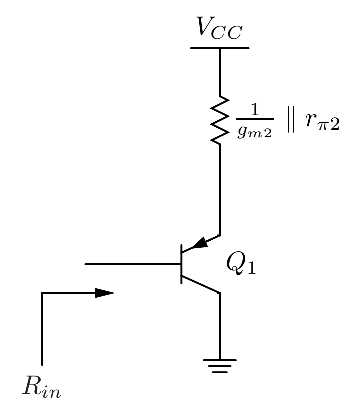
$$R_{in} = \frac{v_t}{i_t} = \boxed{\frac{1}{g_{m1}} \parallel r_{\pi 1}}$$

(c) From our analysis in part (b), we know that looking into the emitter we see a resistance of $\frac{1}{g_{m2}} \parallel r_{\pi 2}$. Thus, we can draw the following equivalent circuit for finding R_{in}:



$$R_{in} = \boxed{r_{\pi 1} + (1+\beta_1)\left(\frac{1}{g_{m2}} \parallel r_{\pi 2}\right)}$$

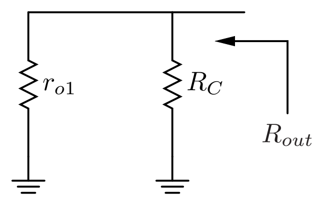
(d) Looking up from the emitter of Q_1 we see an equivalent resistance of $\frac{1}{g_{m2}} \parallel r_{\pi 2}$, so we can draw the following equivalent circuit for finding R_{in}:



$$R_{in} = \boxed{r_{\pi 1} + (1+\beta_1)\left(\frac{1}{g_{m2}} \parallel r_{\pi 2}\right)}$$

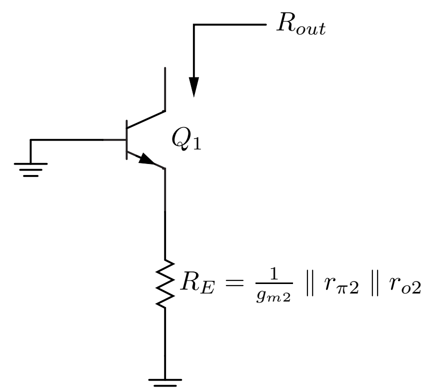
(e) We know that looking into the base of Q_2 we see $R_{in} = \boxed{r_{\pi 2}}$ if the emitter is grounded. Thus, transistor Q_1 does not affect the input impedance of this circuit.

5.6 (a) Looking into the collector of Q_1 we see an equivalent resistance of r_{o1}, so we can draw the following equivalent circuit for finding R_{out}:



$$R_{out} = \boxed{R_C \parallel r_{o1}}$$

(b) Looking into the emitter of Q_2 we see an equivalent resistance of $\frac{1}{g_{m2}} \parallel r_{\pi 2} \parallel r_{o2}$, so we can draw the following equivalent circuit for finding R_{out}:



$$R_{out} = \boxed{r_{o1} + (1 + g_{m1} r_{o1}) \left(r_{\pi 1} \parallel \frac{1}{g_{m2}} \parallel r_{\pi 2} \parallel r_{o2} \right)}$$

5.7 (a)

$$
\begin{aligned}
V_{CC} - I_B(100\ \text{k}\Omega) &= V_{BE} = V_T \ln(I_C/I_S) \\
V_{CC} - \frac{1}{\beta} I_C(100\ \text{k}\Omega) &= V_T \ln(I_C/I_S) \\
I_C &= \boxed{1.754\ \text{mA}} \\
V_{BE} &= V_T \ln(I_C/I_S) = \boxed{746\ \text{mV}} \\
V_{CE} &= V_{CC} - I_C(500\ \Omega) = \boxed{1.62\ \text{V}}
\end{aligned}
$$

Q_1 is operating in forward active.

(b)

$$
\begin{aligned}
I_{E1} &= I_{E2} \Rightarrow V_{BE1} = V_{BE2} \\
V_{CC} - I_{B1}(100\ \text{k}\Omega) &= 2V_{BE1} \\
V_{CC} - \frac{1}{\beta} I_{C1}(100\ \text{k}\Omega) &= 2V_T \ln(I_{C1}/I_S) \\
I_{C1} &= I_{C2} = \boxed{1.035\ \text{mA}} \\
V_{BE1} &= V_{BE2} = \boxed{733\ \text{mV}} \\
V_{CE2} &= V_{BE2} = \boxed{733\ \text{mV}} \\
V_{CE1} &= V_{CC} - I_C(1\ \text{k}\Omega) - V_{CE2} \\
&= \boxed{733\ \text{mV}}
\end{aligned}
$$

Both Q_1 and Q_2 are at the edge of saturation.

(c)

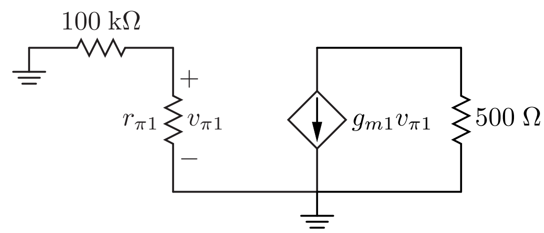
$$
\begin{aligned}
V_{CC} - I_B(100\ \text{k}\Omega) &= V_{BE} + 0.5\ \text{V} \\
V_{CC} - \frac{1}{\beta} I_C(100\ \text{k}\Omega) &= V_T \ln(I_C/I_S) + 0.5\ \text{V} \\
I_C &= \boxed{1.262\ \text{mA}} \\
V_{BE} &= \boxed{738\ \text{mV}} \\
V_{CE} &= V_{CC} - I_C(1\ \text{k}\Omega) - 0.5\ \text{V} \\
&= \boxed{738\ \text{mV}}
\end{aligned}
$$

Q_1 is operating at the edge of saturation.

5.8 See Problem 7 for the derivation of I_C for each part of this problem.

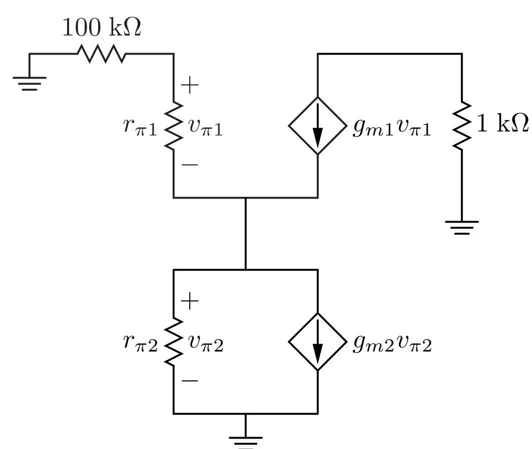
(a)

$$I_{C1} = 1.754 \text{ mA}$$
$$g_{m1} = I_{C1}/V_T = \boxed{67.5 \text{ mS}}$$
$$r_{\pi 1} = \beta/g_{m1} = \boxed{1.482 \text{ k}\Omega}$$



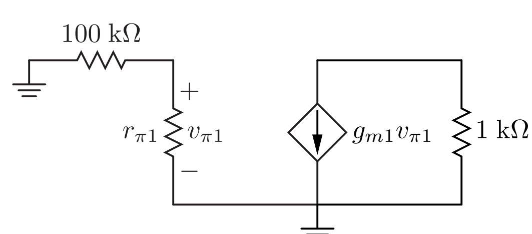
(b)

$$I_{C1} = I_{C2} = 1.034 \text{ mA}$$
$$g_{m1} = g_{m2} = I_{C1}/V_T = \boxed{39.8 \text{ mS}}$$
$$r_{\pi 1} = r_{\pi 2} = \beta/g_{m1} = \boxed{2.515 \text{ k}\Omega}$$



(c)

$$I_{C1} = 1.26 \text{ mA}$$
$$g_{m1} = I_{C1}/V_T = \boxed{48.5 \text{ mS}}$$
$$r_{\pi 1} = \beta/g_{m1} = \boxed{2.063 \text{ k}\Omega}$$



5.9 (a)

$$\begin{aligned}
\frac{V_{CC} - V_{BE}}{34\ \text{k}\Omega} - \frac{V_{BE}}{16\ \text{k}\Omega} &= I_B = \frac{I_C}{\beta} \\
I_C &= \beta\frac{V_{CC} - V_T \ln(I_C/I_S)}{34\ \text{k}\Omega} - \beta\frac{V_T \ln(I_C/I_S)}{16\ \text{k}\Omega} \\
I_C &= \boxed{677\ \mu\text{A}} \\
V_{BE} &= \boxed{726\ \text{mV}} \\
V_{CE} &= V_{CC} - I_C(3\ \text{k}\Omega) = \boxed{468\ \text{mV}}
\end{aligned}$$

Q_1 is in soft saturation.

(b)

$$\begin{aligned}
I_{E1} &= I_{E2} \\
\Rightarrow I_{C1} &= I_{C2} \\
\Rightarrow V_{BE1} &= V_{BE2} = V_{BE} \\
\frac{V_{CC} - 2V_{BE}}{9\ \text{k}\Omega} - \frac{2V_{BE}}{16\ \text{k}\Omega} &= I_{B1} = \frac{I_{C1}}{\beta} \\
I_{C1} &= \beta\frac{V_{CC} - 2V_T \ln(I_{C1}/I_S)}{9\ \text{k}\Omega} - \beta\frac{2V_T \ln(I_{C1}/I_S)}{16\ \text{k}\Omega} \\
I_{C1} &= I_{C2} = \boxed{1.72\ \text{mA}} \\
V_{BE1} &= V_{BE2} = V_{CE2} = \boxed{751\ \text{mV}} \\
V_{CE1} &= V_{CC} - I_{C1}(500\ \Omega) - V_{CE2} = \boxed{890\ \text{mV}}
\end{aligned}$$

Q_1 is in forward active and Q_2 is on the edge of saturation.

(c)

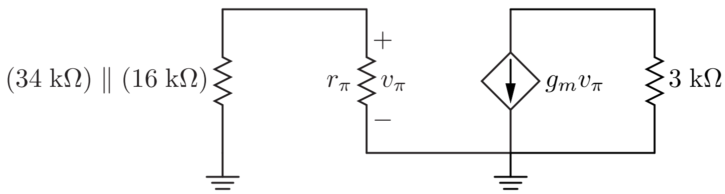
$$\begin{aligned}
\frac{V_{CC} - V_{BE} - 0.5\ \text{V}}{12\ \text{k}\Omega} - \frac{V_{BE} + 0.5\ \text{V}}{13\ \text{k}\Omega} &= I_B = \frac{I_C}{\beta} \\
I_C &= \beta\frac{V_{CC} - V_T \ln(I_C/I_S) - 0.5\ \text{V}}{12\ \text{k}\Omega} - \beta\frac{V_T \ln(I_C/I_S) + 0.5\ \text{V}}{13\ \text{k}\Omega} \\
I_C &= \boxed{1.01\ \text{mA}} \\
V_{BE} &= \boxed{737\ \text{mV}} \\
V_{CE} &= V_{CC} - I_C(1\ \text{k}\Omega) - 0.5\ \text{V} = \boxed{987\ \text{mV}}
\end{aligned}$$

Q_1 is in forward active.

5.10 See Problem 9 for the derivation of I_C for each part of this problem.

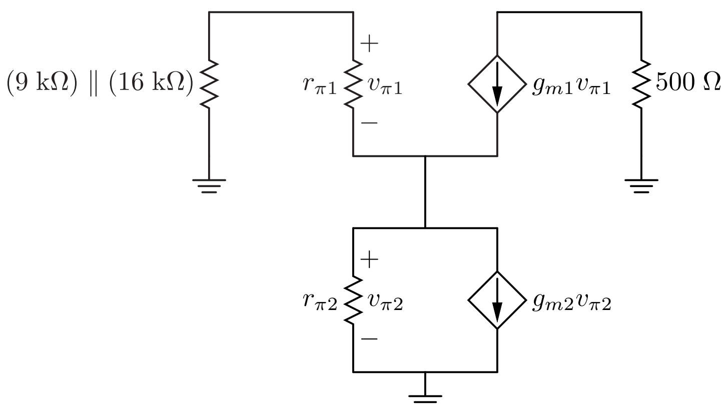
(a)

$$I_C = 677\ \mu\text{A}$$
$$g_m = I_C/V_T = \boxed{26.0\ \text{mS}}$$
$$r_\pi = \beta/g_m = \boxed{3.84\ \text{k}\Omega}$$



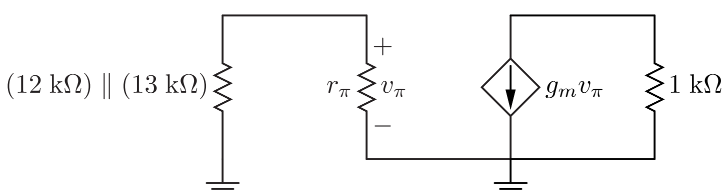
(b)

$$I_{C1} = I_{C2} = 1.72\ \text{mA}$$
$$g_{m1} = g_{m2} = I_{C1}/V_T = \boxed{66.2\ \text{mS}}$$
$$r_{\pi 1} = r_{\pi 2} = \beta/g_{m1} = \boxed{1.51\ \text{k}\Omega}$$



(c)

$$I_C = 1.01\ \text{mA}$$
$$g_m = I_C/V_T = \boxed{38.8\ \text{mS}}$$
$$r_\pi = \beta/g_m = \boxed{2.57\ \text{k}\Omega}$$



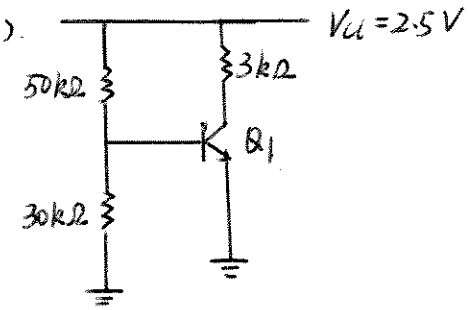
5.11 (a)

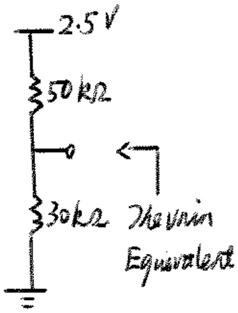
$$
\begin{aligned}
V_{CE} &\geq V_{BE} \text{ (in order to guarantee operation in the active mode)} \\
V_{CC} - I_C(2\text{ k}\Omega) &\geq V_{BE} \\
V_{CC} - I_C(2\text{ k}\Omega) &\geq V_T \ln(I_C/I_S) \\
I_C &\leq 886\ \mu\text{A} \\
\frac{V_{CC} - V_{BE}}{R_B} - \frac{V_{BE}}{3\text{ k}\Omega} &= I_B = \frac{I_C}{\beta} \\
\frac{V_{CC} - V_T \ln(I_C/I_S)}{R_B} - \frac{V_T \ln(I_C/I_S)}{3\text{ k}\Omega} &= \frac{I_C}{\beta} \\
R_B\left(\frac{I_C}{\beta} + \frac{V_T \ln(I_C/I_S)}{3\text{ k}\Omega}\right) &= V_{CC} - V_T \ln(I_C/I_S) \\
R_B &= \frac{V_{CC} - V_T \ln(I_C/I_S)}{\frac{I_C}{\beta} + \frac{V_T \ln(I_C/I_S)}{3\text{ k}\Omega}} \\
R_B &\geq \boxed{7.04\text{ k}\Omega}
\end{aligned}
$$

(b)

$$
\begin{aligned}
\frac{V_{CC} - V_{BE}}{R_B} - \frac{V_{BE}}{3\text{ k}\Omega} &= I_B = \frac{I_C}{\beta} \\
I_C &= \beta\frac{V_{CC} - V_T \ln(I_C/I_S)}{R_B} - \beta\frac{V_T \ln(I_C/I_S)}{3\text{ k}\Omega} \\
I_C &= 1.14\text{ mA} \\
V_{BE} &= 735\text{ mV} \\
V_{CE} &= V_{CC} - I_C(2\text{ k}\Omega) = 215\text{ mV} \\
V_{BC} &= V_{BE} - V_{CE} = \boxed{520\text{ mV}}
\end{aligned}
$$

12). a).





$V_{th} = 2.5 \times \frac{30}{50+30}$
$= 0.9375$

$R_{th} = \frac{30 \times 50}{30+50} k$
$= 18.75k$

Since $I_C = 0.5mA$, $I_B = \frac{I_C}{\beta} = 0.005mA$.

$I_B = \frac{V_{th} - V_{BE}}{R_{th}} \Rightarrow V_{BE} = V_{th} - I_B \cdot R_{th} = 0.84375$

$I_C = I_S \, e^{(\frac{V_{BE}}{V_T})} \Rightarrow I_S = \frac{I_C}{e^{(V_{BE}/V_T)}} = 4.03 \times 10^{-15} (mA)$

b). At the edge of saturation means $V_{BE} - V_{CE} = 0$.

(soft saturation not allowed)

$V_{CE} = 2.5 - I_C \cdot (3k)$, in which $I_C = \beta I_B = \beta (\frac{V_{th} - V_{BE}}{R_{th}})$

so $V_{BE} = 2.5 - \beta (\frac{V_{th} - V_{BE}}{R_{th}}) \cdot (3k)$

Solve this equation:

$V_{BE} = 0.83$.

$I_S = \frac{I_C}{e^{(V_{BE}/V_T)}} = \frac{\beta \, (\frac{V_{th} - V_{BE}}{R_{th}})}{e^{(V_{BE}/V_T)}} = 7.84 \times 10^{-15} (mA)$

5.13 We know the input resistance is $R_{in} = R_1 \parallel R_2 \parallel r_\pi$. Since we want the minimum values of R_1 and R_2 such that $R_{in} > 10\ \text{k}\Omega$, we should pick the maximum value allowable for r_π, which means picking the minimum value allowable for g_m (since $r_\pi \propto 1/g_m$), which is $g_m = 1/260$ S.

$$
\begin{aligned}
g_m &= \frac{1}{260}\ \text{S} \\
I_C &= g_m V_T = 100\ \mu\text{A} \\
V_{BE} &= V_T \ln(I_C/I_S) = 760\ \text{mV} \\
I_B &= \frac{I_C}{\beta} = 1\ \mu\text{A} \\
\frac{V_{CC} - V_{BE}}{R_1} - \frac{V_{BE}}{R_2} &= I_B \\
R_1 &= \frac{V_{CC} - V_{BE}}{I_B + \frac{V_{BE}}{R_2}} \\
r_\pi &= \frac{\beta}{g_m} = 26\ \text{k}\Omega \\
R_{in} &= R_1 \parallel R_2 \parallel r_\pi \\
&= \left(\frac{V_{CC} - V_{BE}}{I_B + \frac{V_{BE}}{R_2}} \right) \parallel R_2 \parallel r_\pi \\
&> 10\ \text{k}\Omega \\
R_2 &> \boxed{23.57\ \text{k}\Omega} \\
R_1 &> \boxed{52.32\ \text{k}\Omega}
\end{aligned}
$$

5.14

$$
\begin{aligned}
g_m &= \frac{I_C}{V_T} \geq \frac{1}{26}\ \text{S} \\
r_\pi &= \frac{\beta}{g_m} = 2.6\ \text{k}\Omega \\
R_{in} &= R_1 \parallel R_2 \parallel r_\pi \\
&\leq r_\pi
\end{aligned}
$$

According to the above analysis, R_{in} cannot be greater than 2.6 kΩ. This means that the requirement that $R_{in} \geq 10$ kΩ cannot be met. Qualitatively, the requirement for g_m to be large forces r_π to be small, and since R_{in} is bounded by r_π, it puts an upper bound on R_{in} that, in this case, is below the required 10 kΩ.

5.15

$$
\begin{aligned}
R_{out} &= R_C = R_0 \\
A_v &= -g_m R_C = -g_m R_0 = -\frac{I_C}{V_T} R_0 = -A_0 \\
I_C &= \frac{A_0}{R_0} V_T \\
r_\pi &= \beta \frac{V_T}{I_C} = \beta \frac{R_0}{A_0} \\
V_{BE} &= V_T \ln(I_C/I_S) = V_T \ln\left(\frac{A_0 V_T}{R_0 I_S}\right) \\
\frac{V_{CC} - V_{BE}}{R_1} - \frac{V_{BE}}{R_2} &= I_B = \frac{I_C}{\beta} \\
R_1 &= \frac{V_{CC} - V_{BE}}{\frac{I_C}{\beta} + \frac{V_{BE}}{R_2}} \\
R_{in} &= R_1 \parallel R_2 \parallel r_\pi \\
&= \left(\frac{V_{CC} - V_T \ln\left(\frac{A_0 V_T}{R_0 I_S}\right)}{\frac{I_C}{\beta} + \frac{V_T}{R_2} \ln\left(\frac{A_0 V_T}{R_0 I_S}\right)}\right) \parallel R_2 \parallel \beta \frac{R_0}{A_0}
\end{aligned}
$$

In order to maximize R_{in}, we can let $R_2 \to \infty$. This gives us

$$
R_{in,max} = \boxed{\left(\beta \frac{V_{CC} - V_T \ln\left(\frac{A_0 V_T}{R_0 I_S}\right)}{I_C}\right) \parallel \beta \frac{R_0}{A_0}}
$$

5.16 (a)

$$I_C = 0.25\ \text{mA}$$
$$V_{BE} = 696\ \text{mV}$$
$$\frac{V_{CC} - V_{BE} - I_E R_E}{R_1} - \frac{V_{BE} + I_E R_E}{R_2} = I_B = \frac{I_C}{\beta}$$
$$R_1 = \frac{V_{CC} - V_{BE} - \frac{1+\beta}{\beta} I_C R_E}{\frac{I_C}{\beta} + \frac{V_{BE} + \frac{1+\beta}{\beta} I_C R_E}{R_2}}$$
$$= \boxed{22.74\ \text{k}\Omega}$$

(b) First, consider a 5 % increase in R_E.

$$R_E = 210\ \Omega$$
$$\frac{V_{CC} - V_{BE} - I_E R_E}{R_1} - \frac{V_{BE} + I_E R_E}{R_2} = I_B = \frac{I_C}{\beta}$$
$$\frac{V_{CC} - V_T \ln(I_C/I_S) - \frac{1+\beta}{\beta} I_C R_E}{R_1} - \frac{V_T \ln(I_C/I_S) + \frac{1+\beta}{\beta} I_C R_E}{R_2} = I_B = \frac{I_C}{\beta}$$
$$I_C = 243\ \mu\text{A}$$
$$\frac{I_C - I_{C,nom}}{I_{C,nom}} \times 100 = \boxed{-2.6\ \%}$$

Now, consider a 5 % decrease in R_E.

$$R_E = 190\ \Omega$$
$$I_C = 257\ \mu\text{A}$$
$$\frac{I_C - I_{C,nom}}{I_{C,nom}} \times 100 = \boxed{+2.8\ \%}$$

5.17

$$
\begin{aligned}
V_{CE} &\geq V_{BE} \text{ (in order to guarantee operation in the active mode)} \\
V_{CC} - I_C R_C &\geq V_T \ln(I_C/I_S) \\
I_C &\leq 833\ \mu\text{A} \\
\frac{V_{CC} - V_{BE} - I_E R_E}{30\ \text{k}\Omega} - \frac{V_{BE} + I_E R_E}{R_2} &= I_B = \frac{I_C}{\beta} \\
R_2 &= \frac{V_{BE} + I_E R_E}{\frac{V_{CC} - V_{BE} - I_E R_E}{30\ \text{k}\Omega} - \frac{I_C}{\beta}} \\
&= \frac{V_T \ln(I_C/I_S) + \frac{1+\beta}{\beta} I_C R_E}{\frac{V_{CC} - V_T \ln(I_C/I_S) - \frac{1+\beta}{\beta} I_C R_E}{30\ \text{k}\Omega} - \frac{I_C}{\beta}} \\
R_2 &\leq \boxed{20.66\ \text{k}\Omega}
\end{aligned}
$$

5.18 (a) First, note that $V_{BE1} = V_{BE2} = V_{BE}$, but since $I_{S1} = 2I_{S2}$, $I_{C1} = 2I_{C2}$. Also note that $\beta_1 = \beta_2 = \beta = 100$.

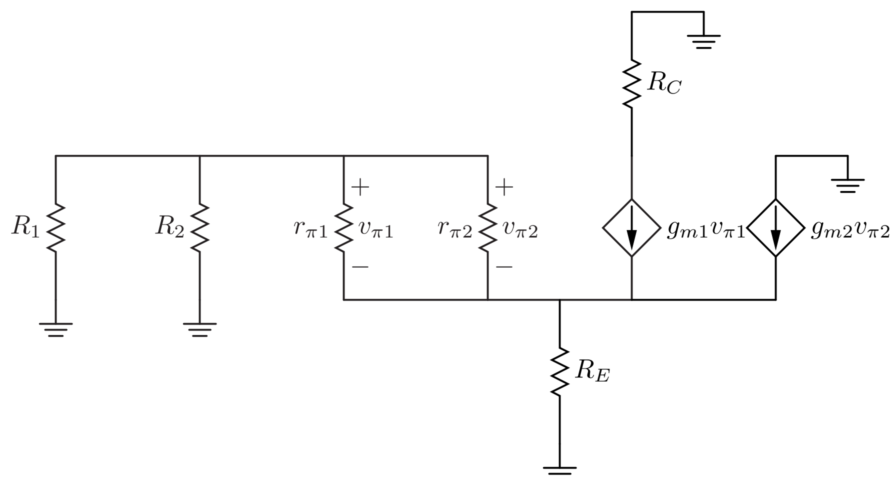
$$I_{B1} = \frac{I_{C1}}{\beta} = \frac{V_{CC} - V_{BE} - (I_{E1} + I_{E2})R_E}{R_1} - \frac{V_{BE} + (I_{E1} + I_{E2})R_E}{R_2}$$

$$I_{C1} = \beta \frac{V_{CC} - V_T \ln(I_{C1}/I_{S1}) - \frac{3}{2}\frac{1+\beta}{\beta} I_{C1} R_E}{R_1} - \frac{V_T \ln(I_{C1}/I_{S1}) + \frac{3}{2}\frac{1+\beta}{\beta} I_{C1} R_E}{R_2}$$

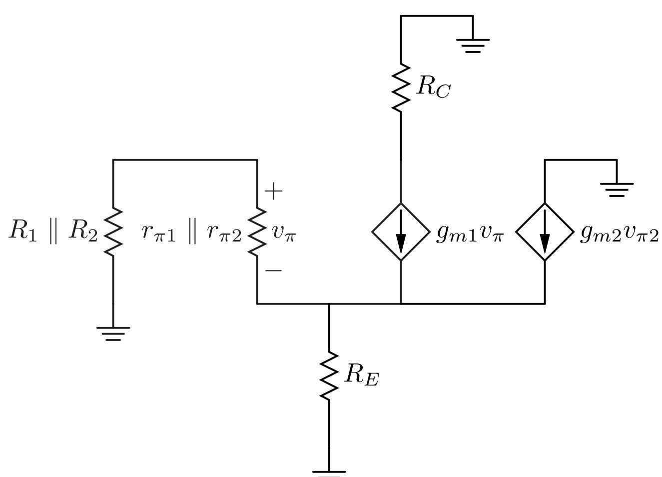
$$I_{C1} = \boxed{707\ \mu\text{A}}$$

$$I_{C2} = \frac{I_{C1}}{2} = \boxed{354\ \mu\text{A}}$$

(b) The small-signal model is shown below.



We can simplify the small-signal model as follows:



$$g_{m1} = I_{C1}/V_T = \boxed{27.2 \text{ mS}}$$
$$r_{\pi 1} = \beta_1/g_{m1} = \boxed{3.677 \text{ k}\Omega}$$
$$g_{m2} = I_{C2}/V_T = \boxed{13.6 \text{ mS}}$$
$$r_{\pi 2} = \beta_2/g_{m2} = \boxed{7.355 \text{ k}\Omega}$$

5.19 (a)

$$I_{E1} = I_{E2} \Rightarrow V_{BE1} = V_{BE2}$$

$$\frac{V_{CC} - 2V_{BE1}}{9\text{ k}\Omega} - \frac{2V_{BE1}}{16\text{ k}\Omega} = I_{B1} = \frac{I_{C1}}{\beta_1}$$

$$I_{C1} = \beta_1 \frac{V_{CC} - 2V_T \ln(I_{C1}/I_{S1})}{9\text{ k}\Omega} - \beta_1 \frac{2V_T \ln(I_{C1}/I_{S1})}{16\text{ k}\Omega}$$

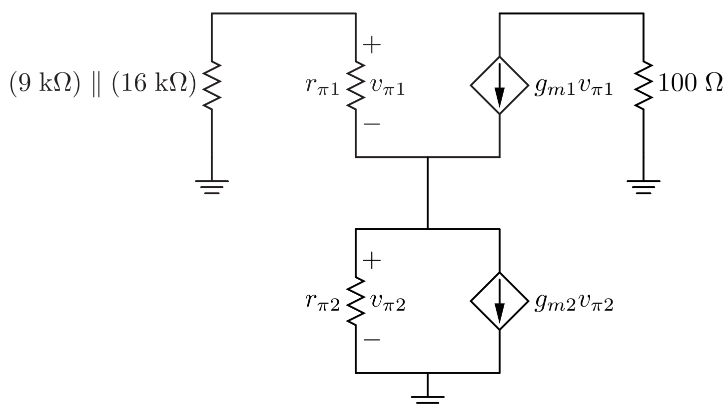
$$I_{C1} = I_{C2} = \boxed{1.588\text{ mA}}$$

$$V_{BE1} = V_{BE2} = V_T \ln(I_{C1}/I_{S1}) = \boxed{754\text{ mV}}$$

$$V_{CE2} = V_{BE2} = \boxed{754\text{ mV}}$$

$$V_{CE1} = V_{CC} - I_{C1}(100\ \Omega) - V_{CE2} = \boxed{1.587\text{ V}}$$

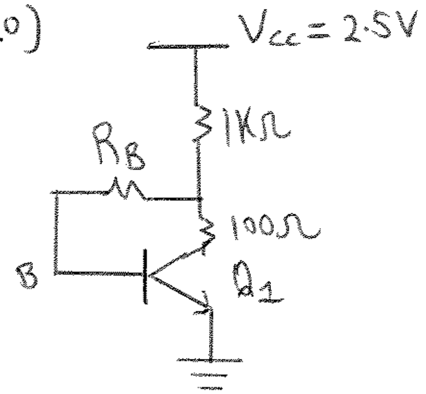
(b) The small-signal model is shown below.



$$g_{m1} = g_{m2} = \frac{I_{C1}}{V_T} = \boxed{61.1\text{ mS}}$$

$$r_{\pi 1} = r_{\pi 2} = \frac{\beta_1}{g_{m1}} = \boxed{1.637\text{ k}\Omega}$$

20)



$I_c = 1mA$

$V_{BE} = V_T \ln\left(\frac{I_c}{I_s}\right) = 0.750V$

$V_B = 2.5 - (I_E(1K\Omega) + I_B R_B) = 0.750V$

$I_E = 1.01mA$

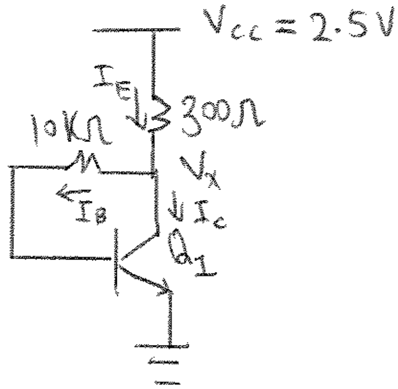
$I_B = 0.01mA$

$V_B = 2.5 - 1.01 - 0.01 R_B = 0.750$

$0.74 = 0.01 R_B$

$R_B = 74K\Omega$

21)



$V_X = 1.1V$

$\beta = 100$

$I_s = ?$

$I_E = I_B + I_C$

$I_E = \frac{2.5 - 1.1}{300\Omega} = 4.67\,mA$

$I_B = \frac{I_c}{\beta}$

$I_E = \frac{I_c}{\beta} + I_c = 4.67\,mA$

$I_c = 4.624\,mA$

$I_s = \frac{I_c}{e^{\left(\frac{V_{BE}}{V_T}\right)}}$, $V_{BE} = 1.1 - \frac{4.624}{100}(10K) = 0.6376V$

$I_s = 1.035 \times 10^{-10}\,mA$

$I_s = 1.035 \times 10^{-13}\,A$

5.22

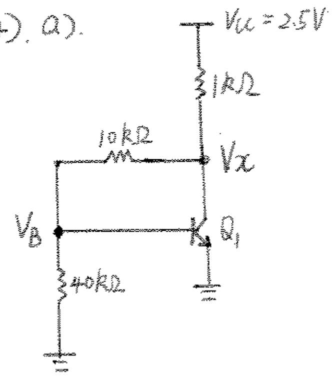
$$
\begin{aligned}
V_{CC} - I_E(500\ \Omega) - I_B(20\ \text{k}\Omega) - I_E(400\ \Omega) &= V_{BE} \\
V_{CC} - \frac{1+\beta}{\beta} I_C(500\ \Omega + 400\ \Omega) - \frac{1}{\beta} I_C(20\ \text{k}\Omega) &= V_T \ln(I_C/I_S) \\
I_C &= \boxed{1.584\ \text{mA}} \\
V_{BE} &= V_T \ln(I_C/I_S) = \boxed{754\ \text{mV}} \\
V_{CE} &= V_{CC} - I_E(500\ \Omega) - I_E(400\ \Omega) \\
&= V_{CC} - \frac{1+\beta}{\beta} I_C(500\ \Omega + 400\ \Omega) = \boxed{1.060\ \text{V}}
\end{aligned}
$$

Q_1 is operating in forward active.

5.23

$$\begin{aligned}
V_{BC} &\leq 200 \text{ mV} \\
V_{CC} - I_E(1 \text{ k}\Omega) - I_B R_B - (V_{CC} - I_E(1 \text{ k}\Omega) - I_C(500 \ \Omega)) &\leq 200 \text{ mV} \\
I_C(500 \ \Omega) - I_B R_B &\leq 200 \text{ mV} \\
I_B R_B &\geq I_C(500 \ \Omega) - 200 \text{ mV} \\
V_{CC} - I_E(1 \text{ k}\Omega) - I_B R_B &= V_{BE} = V_T \ln(I_C/I_S) \\
V_{CC} - \frac{1+\beta}{\beta} I_C(1 \text{ k}\Omega) - I_C(500 \ \Omega) + 200 \text{ mV} &\leq V_T \ln(I_C/I_S) \\
I_C &\geq 1.29 \text{ mA} \\
R_B &\geq \frac{I_C(500 \ \Omega) - 200 \text{ mV}}{\frac{I_C}{\beta}} \\
&\geq \boxed{34.46 \text{ k}\Omega}
\end{aligned}$$

24). a).



$I_S = 8 \times 10^{-16} A$

$\beta = 100$

$V_A = \infty$

$V_x = 2.5 - \left(\frac{I_c}{\alpha} + \frac{V_B}{40k}\right) \cdot 1k$

$V_x = \left(\frac{V_B}{40k} + I_B\right) 10k + V_B = \left(\frac{V_B}{40k} + \frac{I_c}{\beta}\right) 10k + V_B$

Equating V_x $\Rightarrow$ $2.5 - \left(V_B + \frac{V_B \cdot 1k}{40k} + \frac{V_B \cdot 10k}{40k}\right) = \frac{I_c}{\alpha} \cdot 1k + \frac{I_c}{\beta} \cdot 10k$

$\Rightarrow I_c = \dfrac{2.5 - 1.275 V_B}{\frac{1k}{\alpha} + \frac{10k}{\beta}}$

Guess $V_B = 0.8$

$$I_C = \frac{1.48}{\frac{1k}{0.99} + \frac{10k}{100}} = 1.33\, m(A)$$

Then

$$V_B = V_T \ln\left(\frac{I_c}{I_S}\right) = 0.732, \text{ not } 0.8$$

Reiterate

$$I_C = \frac{1.3667}{1.11} = 1.4113\, (mA)$$

$$V_B = V_T \ln\left(\frac{I_c}{I_S}\right) = 0.733$$

So V_B converges to 0.73V

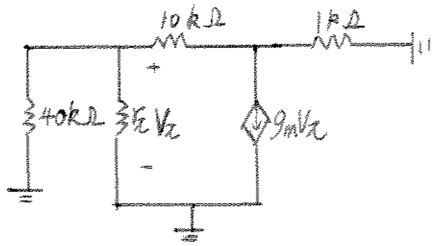
$I_C = 1.41\, mA$

$I_B = 14.1\, \mu A$

$V_{CE} = 2.5V - \left(\frac{1.41}{0.99} + \frac{0.73}{40}\right) \times 1V = 1.06V$

$V_{BE} = 0.73V$

24 b) Small Signal



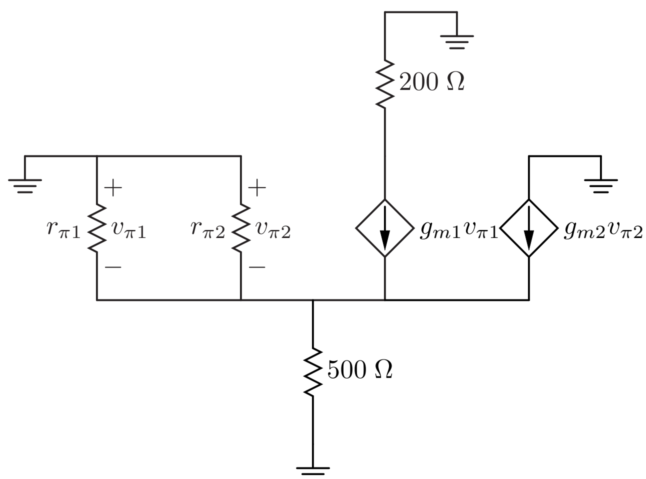
$$g_m = \frac{I_C}{V_T} = 0.054\ S$$

$$r_\pi = \frac{\beta}{g_m} = 1844\Omega$$

5.25 (a)

$$
\begin{aligned}
I_{C1} &= 1 \text{ mA} \\
V_{CC} - (I_{E1} + I_{E2})(500\ \Omega) &= V_T \ln(I_{C2}/I_{S2}) \\
V_{CC} - \left(\frac{1+\beta}{\beta} I_{C1} + \frac{1+\beta}{\beta} I_{C2}\right)(500\ \Omega) &= V_T \ln(I_{C2}/I_{S2}) \\
I_{C2} &= 2.42 \text{ mA} \\
V_B - (I_{E1} + I_{E2})(500\ \Omega) &= V_T \ln(I_{C1}/I_{S1}) \\
V_B - \left(\frac{1+\beta}{\beta} I_{C1} + \frac{1+\beta}{\beta} I_{C2}\right)(500\ \Omega) &= V_T \ln(I_{C1}/I_{S1}) \\
V_B &= \boxed{2.68 \text{ V}}
\end{aligned}
$$

(b) The small-signal model is shown below.



$$
\begin{aligned}
g_{m1} &= I_{C1}/V_T = \boxed{38.5 \text{ mS}} \\
r_{\pi 1} &= \beta_1/g_{m1} = \boxed{2.6 \text{ k}\Omega} \\
g_{m2} &= I_{C2}/V_T = \boxed{93.1 \text{ mS}} \\
r_{\pi 2} &= \beta_2/g_{m2} = \boxed{1.074 \text{ k}\Omega}
\end{aligned}
$$

5.26 (a)

$$V_{CC} - I_B(60\text{ k}\Omega) = V_{EB}$$
$$V_{CC} - \frac{1}{\beta_{pnp}} I_C(60\text{ k}\Omega) = V_T \ln(I_C/I_S)$$
$$I_C = \boxed{1.474\text{ mA}}$$
$$V_{EB} = V_T \ln(I_C/I_S) = \boxed{731\text{ mV}}$$
$$V_{EC} = V_{CC} - I_C(200\ \Omega) = \boxed{2.205\text{ V}}$$

Q_1 is operating in forward active.

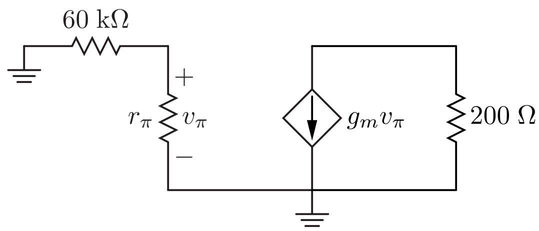
(b)

$$V_{CC} - V_{BE1} - I_{B2}(80\text{ k}\Omega) = V_{EB2}$$
$$V_{CC} - V_T \ln(I_{C1}/I_S) - I_{B2}(80\text{ k}\Omega) = V_T \ln(I_{C2}/I_S)$$
$$I_{C1} = \frac{\beta_{npn}}{1+\beta_{npn}} I_{E1}$$
$$= \frac{\beta_{npn}}{1+\beta_{npn}} I_{E2}$$
$$= \frac{\beta_{npn}}{1+\beta_{npn}} \cdot \frac{1+\beta_{pnp}}{\beta_{pnp}} I_{C2}$$
$$V_{CC} - V_T \ln\left(\frac{\beta_{npn}}{1+\beta_{npn}} \cdot \frac{1+\beta_{pnp}}{\beta_{pnp}} \cdot \frac{I_{C2}}{I_S}\right) - \frac{1}{\beta_{pnp}} I_{C2}(80\text{ k}\Omega) = V_T \ln(I_{C2}/I_S)$$
$$I_{C2} = \boxed{674\ \mu\text{A}}$$
$$V_{BE2} = V_T \ln(I_{C2}/I_S) = \boxed{711\text{ mV}}$$
$$I_{C1} = \boxed{680\ \mu\text{A}}$$
$$V_{BE1} = V_T \ln(I_{C1}/I_S) = \boxed{711\text{ mV}}$$
$$V_{CE1} = V_{BE1} = \boxed{711\text{ mV}}$$
$$V_{CE2} = V_{CC} - V_{CE1} - I_{C2}(300\ \Omega)$$
$$= \boxed{1.585\text{ V}}$$

Q_1 is operating on the edge of saturation. Q_2 is operating in forward active.

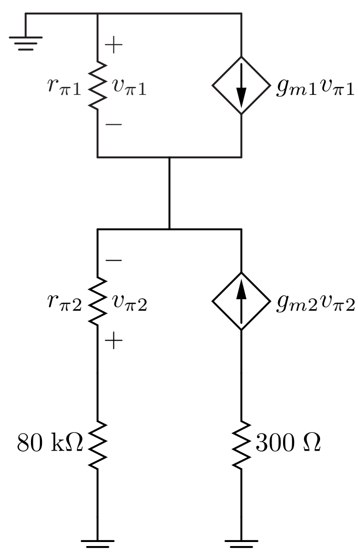
5.27 See Problem 26 for the derivation of I_C for each part of this problem.

(a) The small-signal model is shown below.



$$I_C = 1.474 \text{ mA}$$
$$g_m = \frac{I_C}{V_T} = \boxed{56.7 \text{ mS}}$$
$$r_\pi = \frac{\beta}{g_m} = \boxed{1.764 \text{ k}\Omega}$$

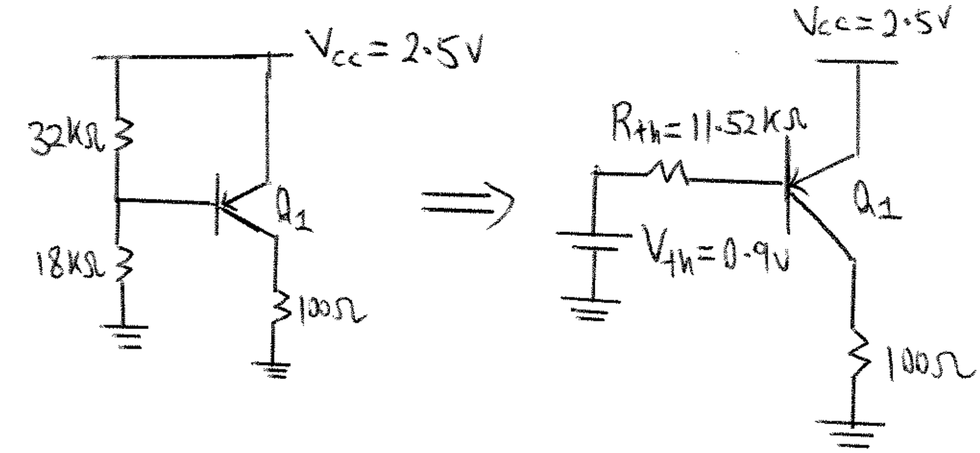
(b) The small-signal model is shown below.



$$I_{C1} = 680 \text{ µA}$$
$$g_{m1} = \frac{I_{C1}}{V_T} = \boxed{26.2 \text{ mS}}$$
$$r_{\pi 1} = \frac{\beta_{npn}}{g_{m1}} = \boxed{3.824 \text{ k}\Omega}$$
$$I_{C2} = 674 \text{ µA}$$
$$g_{m2} = \frac{I_{C2}}{V_T} = \boxed{25.9 \text{ mS}}$$
$$r_{\pi 2} = \frac{\beta_{pnp}}{g_{m2}} = \boxed{1.929 \text{ k}\Omega}$$

28)

a)



$$I_c = \beta_{pnp}\left(\frac{2.5 - |V_{BE}| - V_{th}}{R_{th}}\right)$$

Guess $|V_{BE}| = 0.7\ V$, $I_c = 3.91mA$

$|V_{BE}| = V_T \ln\left(\frac{I_c}{I_s}\right) = 0.757\ V$

Reiterate, $|V_{BE}| = 0.757V$, $I_c = 3.66mA$

$|V_{BE}| = V_T \ln\left(\frac{I_c}{I_s}\right) = 0.755V$

Reiterate, $|V_{BE}| = 0.755V$, $I_c = 3.67mA$

$|V_{BE}| = V_T \ln\left(\frac{I_c}{I_s}\right) = 0.755V$, Converged!!

$V_c = (3.67mA)(0.1K\Omega) = 0.367V$, $V_B \approx 2.5 - 0.755 = 1.745V$

Q_1 in forward active.

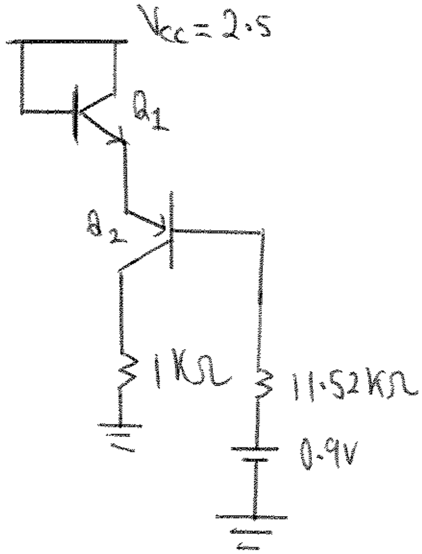
Bias point:

$I_c = 3.67mA$ $\quad |V_{BE}| = 0.755$

$I_B = 73.4\ \mu A$ $\quad |V_{CE}| = 2.5 - 0.367 = 2.133V$

28)

b)



$$I_{C2} = \left(\frac{2.5-(V_{BE1}+V_{BE2})-0.9}{11.52\,K}\right)50$$

$$I_{C1} = I_{C2}(1.0099)$$

(From β relation)

$$V_{BE1} = V_T \ln\left(\frac{I_{C1}}{I_S}\right)$$

$$|V_{BE2}| = V_T \ln\left(\frac{I_{C2}}{I_S}\right)$$

Guess, $V_{BE1} = V_{BE2} = 0.7V$

$I_{C2} = 0.868\,mA$, $I_{C1} = 0.877\,mA$

$V_{BE1} = V_T \ln\left(\frac{I_{C1}}{I_S}\right) = 0.718V$, $|V_{BE2}| = V_T \ln\left(\frac{I_{C2}}{I_S}\right) = 0.717$

Reiterate, $V_{BE1} = 0.718V$, $|V_{BE2}| = 0.717V$

$I_{C2} = 0.716\,mA$, $I_{C1} = 0.723\,mA$

$V_{BE1} = V_T \ln\left(\frac{I_{C1}}{I_S}\right) = 0.713V$, $|V_{BE2}| = V_T \ln\left(\frac{I_{C2}}{I_S}\right) = 0.712V$

Reiterate, $V_{BE1} = 0.713V$, $|V_{BE2}| = 0.712V$

$I_{C2} = 0.760\,mA$, $I_{C1} = 0.767\,mA$

$V_{BE1} = 0.714V$, $|V_{BE2}| = 0.714V$

28)

b)

Reiterate, $V_{BE_1} = 0.714V$, $|V_{BE_2}| = 0.714V$

$I_{C_2} = 0.747mA$, $I_{C_1} = 0.754mA$

$V_{BE_1} = V_T \ln\left(\frac{I_c}{I_s}\right) = 0.714V,$

$|V_{BE_2}| = 0.714\ V$

$V_{B_2} = \frac{(0.747mA)}{50}(11.52K\Omega) + 0.9 = 1.07\ V$

$V_{C_2} = (0.747mA)(1K\Omega) = 0.747\ V$

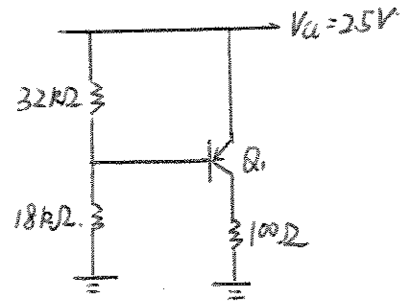
Q_2 is in forward-active region. Q_1 is always in forward-active region.

Bias point:

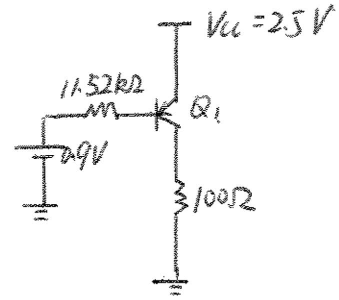
$V_{BE_1} = 0.714V$	$\lvert V_{BE_2}\rvert = 0.714V$
$I_{C_1} = 0.754\ mA$	$I_{C2} = 0.747mA$
$I_{B_1} = 7.54\mu A$	$I_{B2} = 14.94\ \mu A$
$V_{CE_1} = 0.714V$	$\lvert V_{CE_2}\rvert = 2.5 - 0.714 - 0.747 = 1.039V$

29)

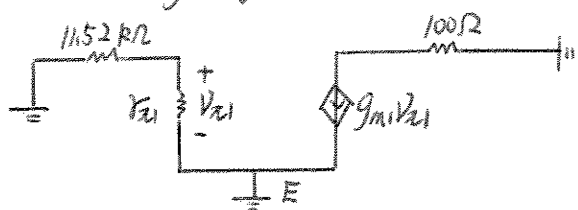
a).



⟹



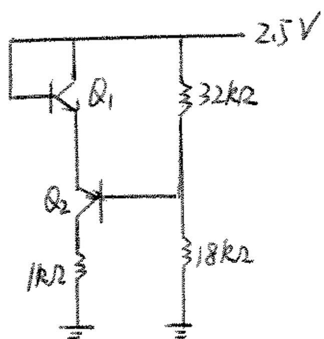
Small Signal:



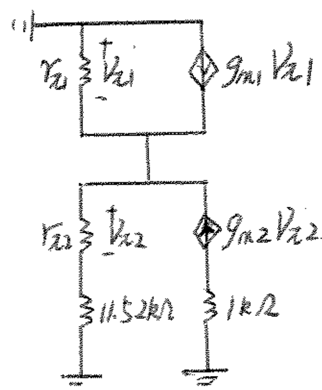
$$g_{m1} = \frac{3.67mA}{26mV} = 0.141\ S$$

$$r_\pi = \frac{50}{0.141}\ \Omega = 354.2\ \Omega$$

b).



⟹



$g_{m1} = 0.029\ S$

$r_{\pi 1} = 3448.3\ \Omega$

$g_{m2} = 0.0287\ S$

$r_{\pi 2} = 1740.3\ \Omega$.

5.30

$$\begin{aligned}
V_{CC} - I_C(1\text{ k}\Omega) &= V_{EC} = V_{EB} \text{ (in order for } Q_1 \text{ to operate at the edge of saturation)} \\
&= V_T \ln(I_C/I_S) \\
I_C &= 1.761\text{ mA} \\
V_{EB} &= 739\text{ mV} \\
\frac{V_{CC} - V_{EB}}{R_B} - \frac{V_{EB}}{5\text{ k}\Omega} &= I_B = \frac{I_C}{\beta} \\
R_B &= 9.623\text{ k}\Omega
\end{aligned}$$

First, let's consider when R_B is 5 % larger than its nominal value.

$$\begin{aligned}
R_B &= 10.104\text{ k}\Omega \\
\frac{V_{CC} - V_T \ln(I_C/I_S)}{R_B} - \frac{V_T \ln(I_C/I_S)}{5\text{ k}\Omega} &= \frac{I_C}{\beta} \\
I_C &= 1.411\text{ mA} \\
V_{EB} &= 733\text{ mV} \\
V_{EC} &= V_{CC} - I_C(1\text{ k}\Omega) = 1.089\text{ V} \\
V_{CB} &= \boxed{-355\text{ mV}} \text{ (the collector-base junction is reverse biased)}
\end{aligned}$$

Now, let's consider when R_B is 5 % smaller than its nominal value.

$$\begin{aligned}
R_B &= 9.142\text{ k}\Omega \\
\frac{V_{CC} - V_T \ln(I_C/I_S)}{R_B} - \frac{V_T \ln(I_C/I_S)}{5\text{ k}\Omega} &= \frac{I_C}{\beta} \\
I_C &= 2.160\text{ mA} \\
V_{EB} &= 744\text{ mV} \\
V_{EC} &= V_{CC} - I_C(1\text{ k}\Omega) = 340\text{ mV} \\
V_{CB} &= \boxed{405\text{ mV}} \text{ (the collector-base junction is forward biased)}
\end{aligned}$$

5.31

$$\frac{V_{BC} + I_C(5\ \text{k}\Omega)}{10\ \text{k}\Omega} - \frac{V_{CC} - V_{BC} - I_C(5\ \text{k}\Omega)}{10\ \text{k}\Omega} = I_B = \frac{I_C}{\beta}$$

$$V_{BC} = 300\ \text{mV}$$

$$I_C = 194\ \mu\text{A}$$

$$V_{EB} = V_T \ln(I_C/I_S) = 682\ \text{mV}$$

$$V_{CC} - I_E R_E - I_C(5\ \text{k}\Omega) = V_{EC} = V_{EB} + 300\ \text{mV}$$

$$V_{CC} - \frac{1+\beta}{\beta} I_C R_E - I_C(5\ \text{k}\Omega) = V_{EB} + 300\ \text{mV}$$

$$R_E = \boxed{2.776\ \text{k}\Omega}$$

Let's look at what happens when R_E is halved.

$$R_E = 1.388\ \text{k}\Omega$$

$$\frac{V_{CC} - I_E R_E - V_{EB}}{10\ \text{k}\Omega} - \frac{V_{CC} - (V_{CC} - I_E R_E - V_{EB})}{10\ \text{k}\Omega} = I_B = \frac{I_C}{\beta}$$

$$\beta \frac{V_{CC} - \frac{1+\beta}{\beta} I_C R_E - V_T \ln(I_C/I_S)}{10\ \text{k}\Omega} - \beta \frac{V_{CC} - \left(V_{CC} - \frac{1+\beta}{\beta} I_C R_E - V_T \ln(I_C/I_S)\right)}{10\ \text{k}\Omega} = I_C$$

$$I_C = 364\ \mu\text{A}$$

$$V_{EB} = 698\ \mu\text{V}$$

$$V_{EC} = 164\ \mu\text{V}$$

Thus, when R_E is halved, Q_1 operates in deep saturation.

5.32

$$V_{CC} - I_B(20\text{ k}\Omega) - I_E(1.6\text{ k}\Omega) = V_{BE} = V_T \ln(I_C/I_S)$$

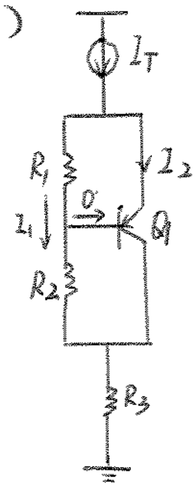
$$V_{CC} - \frac{I_C}{\beta}(20\text{ k}\Omega) - \frac{1+\beta}{\beta} I_C(1.6\text{ k}\Omega) = V_{BE} = V_T \ln(I_C/I_S)$$

$$I_S = \frac{I_C}{e^{\left[V_{CC} - \frac{I_C}{\beta}(20\text{ k}\Omega) - \frac{1+\beta}{\beta} I_C(1.6\text{ k}\Omega)\right]/V_T}}$$

$$I_C = 1\text{ mA}$$

$$I_S = \boxed{3 \times 10^{-14}\text{ A}}$$

33)



If Base current is neglected, $I_C = I_E$

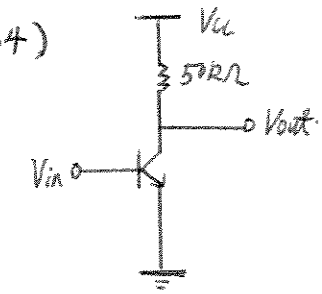
$$I_1 = \frac{V_E - V_C}{R_1 + R_2}$$

$$|V_{BE}| = I_1 R_1 = \frac{V_E - V_C}{R_1 + R_2} R_1 = \frac{|V_{CE}|}{R_1 + R_2} R_1$$

So $$\frac{|V_{CE}|}{|V_{BE}|} = \frac{R_1 + R_2}{R_1}$$

Let $A = \frac{R_1 + R_2}{R_1}$, $|V_{CE}| = A\,|V_{BE}|$, thus $|V_{BE}|$ is multiplied.

34)



$A_V = g_m R_C = 20$

$\frac{I_C R_C}{V_T} = 20 \Rightarrow I_C = \frac{20 V_T}{R_C}$

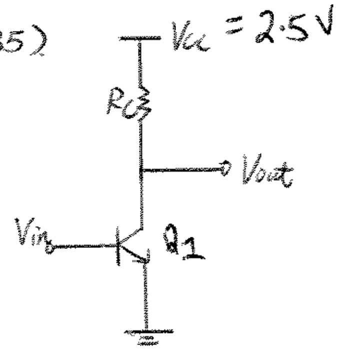
$I_C = 0.0104 mA$

$V_{CC} - (50k\Omega)(0.0104 mA) = V_{BE}$

$\Rightarrow V_{CC} - 50 \times 0.0104 V = 0.8 V$

$\Rightarrow V_{CC} = 1.32 V$

35)



$V_A = 10V$, $r_o = \frac{V_A}{I_C}$, $g_m = \frac{I_c}{V_T}$

$$\left|\frac{V_{out}}{V_{in}}\right| = g_m (R_C // r_o) = g_m \left(\frac{R_C r_o}{R_C + r_o}\right) = \frac{R_C V_A}{V_T (R_C + \frac{V_A}{I_C})}$$

As the equation above shows, a large gain means a large I_c. However, a large I_c will drive Q_1 into saturation. So a tradeoff must be made. The maximum limit for I_c is when it drives Q_1 into the edge of saturation, namely, $V_{BE} = V_{CE}$.

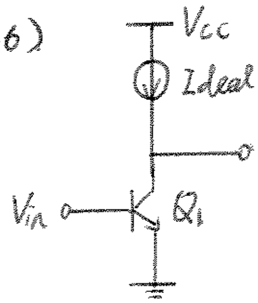
$$V_{CE} = V_{cc} - I_c (1K)$$

$V_{BE} = 0.8V$, $V_{cc} = 2.5V$

$$0.8 = 2.5 - I_c 1K$$

$$I_c = 1.7mA$$

36)



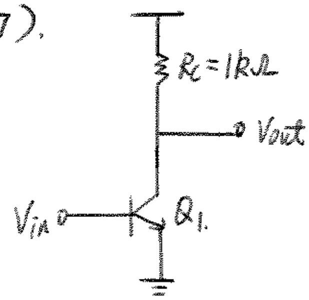
$A_v = 50$

$R_{out} = r_o = 10k\Omega$

$$A_v = g_m R_{out} = \frac{I_c}{V_T} R_{out} = 50$$

$$I_c = 50\left(\frac{V_T}{R_{out}}\right) = 0.13mA$$

37).



$$I_c = I_s \exp\left(\frac{V_{BE}}{2V_T}\right)$$

$$g_m = \frac{\partial I_c}{\partial V_{BE}} = \frac{I_c}{2V_T}$$

$$R_{out} = R_c$$

$$\left|\frac{V_{out}}{V_{in}}\right| = g_m R_{out} = \frac{I_c R_c}{2V_T} = \frac{(1mA)(1K\Omega)}{(2)(0.026V)} = 19.23$$

5.38 (a)

$$A_v = \boxed{-g_{m1}\left(\frac{1}{g_{m2}} \parallel r_{\pi 2}\right)}$$
$$R_{in} = \boxed{r_{\pi 1}}$$
$$R_{out} = \boxed{\frac{1}{g_{m2}} \parallel r_{\pi 2}}$$

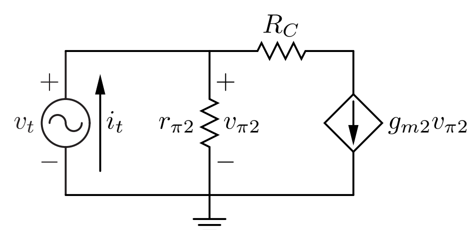
(b)

$$A_v = \boxed{-g_{m1}\left(R_1 + \frac{1}{g_{m2}} \parallel r_{\pi 2}\right)}$$
$$R_{in} = \boxed{r_{\pi 1}}$$
$$R_{out} = \boxed{R_1 + \frac{1}{g_{m2}} \parallel r_{\pi 2}}$$

(c)

$$A_v = \boxed{-g_{m1}\left(R_C + \frac{1}{g_{m2}} \parallel r_{\pi 2}\right)}$$
$$R_{in} = \boxed{r_{\pi 1}}$$
$$R_{out} = \boxed{R_C + \frac{1}{g_{m2}} \parallel r_{\pi 2}}$$

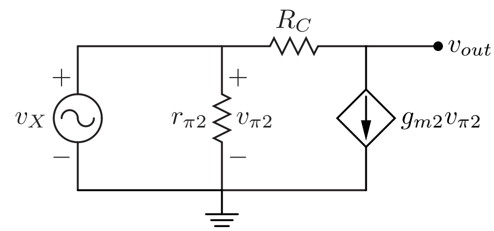
(d) Let's determine the equivalent resistance seen looking up from the output by drawing a small-signal model and applying a test source.



$$i_t = \frac{v_{\pi 2}}{r_{\pi 2}} + g_{m2}v_{\pi 2}$$
$$v_{\pi 2} = v_t$$
$$i_t = v_t\left(\frac{1}{r_{\pi 2}} + g_{m2}\right)$$
$$\frac{v_t}{i_t} = \frac{1}{g_{m2}} \parallel r_{\pi 2}$$

$$A_v = \boxed{-g_{m1}\left(\frac{1}{g_{m2}} \parallel r_{\pi 2}\right)}$$
$$R_{in} = \boxed{r_{\pi 1}}$$
$$R_{out} = \boxed{\frac{1}{g_{m2}} \parallel r_{\pi 2}}$$

(e) From (d), we know the gain from the input to the collector of Q_1 is $-g_{m1}\left(\frac{1}{g_{m2}} \parallel r_{\pi 2}\right)$. If we find the gain from the collector of Q_1 to v_{out}, we can multiply these expressions to find the overall gain. Let's draw the small-signal model to find the gain from the collector of Q_1 to v_{out}. I'll refer to the collector of Q_1 as node X in the following derivation.

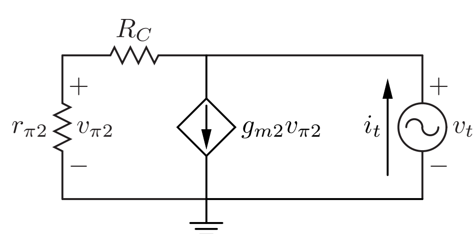


$$\frac{v_X - v_{out}}{R_C} = g_{m2}v_{\pi 2}$$
$$v_{\pi 2} = v_X$$
$$\frac{v_X - v_{out}}{R_C} = g_{m2}v_X$$
$$v_X\left(\frac{1}{R_C} - g_{m2}\right) = \frac{v_{out}}{R_C}$$
$$\frac{v_{out}}{v_X} = 1 - g_{m2}R_C$$

Thus, we have

$$A_v = \boxed{-g_{m1}\left(\frac{1}{g_{m2}} \parallel r_{\pi 2}\right)(1 - g_{m2}R_C)}$$
$$R_{in} = \boxed{r_{\pi 1}}$$

To find the output resistance, let's draw the small-signal model and apply a test source at the output. Note that looking into the collector of Q_1 we see infinite resistance, so we can exclude it from the small-signal model.



$$
\begin{aligned}
i_t &= g_{m2} v_{\pi 2} + \frac{v_{\pi 2}}{r_{\pi 2}} \\
v_{\pi 2} &= \frac{r_{\pi 2}}{r_{\pi 2} + R_C} v_t \\
i_t &= \left(g_{m2} + \frac{1}{r_{\pi 2}} \right) \frac{r_{\pi 2}}{r_{\pi 2} + R_C} v_t \\
R_{out} &= \frac{v_t}{i_t} \\
&= \boxed{\left(\frac{1}{g_{m2}} \parallel r_{\pi 2} \right) \frac{r_{\pi 2} + R_C}{r_{\pi 2}}}
\end{aligned}
$$

5.39 (a)

$$A_v = \boxed{-g_{m1}\left(r_{o1} \parallel \frac{1}{g_{m2}} \parallel r_{\pi 2} \parallel r_{o2}\right)}$$

$$R_{in} = \boxed{r_{\pi 1}}$$

$$R_{out} = \boxed{r_{o1} \parallel \frac{1}{g_{m2}} \parallel r_{\pi 2} \parallel r_{o2}}$$

(b)

$$A_v = \boxed{-g_{m1}\left[r_{o1} \parallel \left(R_1 + \frac{1}{g_{m2}} \parallel r_{\pi 2} \parallel r_{o2}\right)\right]}$$

$$R_{in} = \boxed{r_{\pi 1}}$$

$$R_{out} = \boxed{r_{o1} \parallel \left(R_1 + \frac{1}{g_{m2}} \parallel r_{\pi 2} \parallel r_{o2}\right)}$$

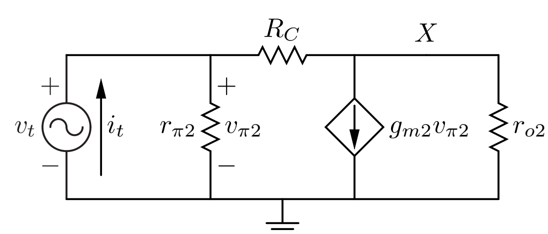
(c)

$$A_v = \boxed{-g_{m1}\left[r_{o1} \parallel \left(R_C + \frac{1}{g_{m2}} \parallel r_{\pi 2} \parallel r_{o2}\right)\right]}$$

$$R_{in} = \boxed{r_{\pi 1}}$$

$$R_{out} = \boxed{r_{o1} \parallel \left(R_C + \frac{1}{g_{m2}} \parallel r_{\pi 2} \parallel r_{o2}\right)}$$

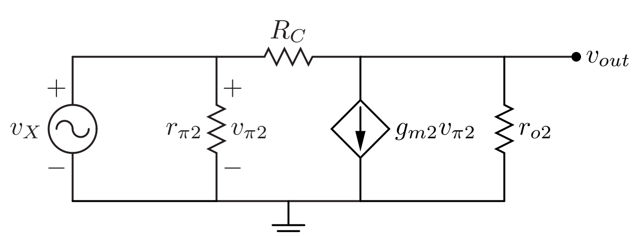
(d) Let's determine the equivalent resistance seen looking up from the output by drawing a small-signal model and applying a test source.



$$
\begin{aligned}
i_t &= \frac{v_{\pi 2}}{r_{\pi 2}} + \frac{v_t - v_X}{R_C} \\
\frac{v_X - v_t}{R_C} + g_{m2} v_{\pi 2} + \frac{v_X}{r_{o2}} &= 0 \\
v_{\pi 2} &= v_t \\
v_X \left(\frac{1}{R_C} + \frac{1}{r_{o2}} \right) &= v_t \left(\frac{1}{R_C} - g_{m2} \right) \\
v_X &= v_t \left(\frac{1}{R_C} - g_{m2} \right) (r_{o2} \parallel R_C) \\
i_t &= \frac{v_t}{r_{\pi 2}} + \frac{v_t}{R_C} - \frac{1}{R_C} v_t \left(\frac{1}{R_C} - g_{m2} \right) (r_{o2} \parallel R_C) \\
&= v_t \left[\frac{1}{r_{\pi 2}} + \frac{1}{R_C} - \frac{1}{R_C} \left(\frac{1}{R_C} - g_{m2} \right) (r_{o2} \parallel R_C) \right] \\
&= v_t \left[\frac{1}{r_{\pi 2}} + \frac{1}{R_C} + \left(g_{m2} - \frac{1}{R_C} \right) \frac{r_{o2}}{r_{o2} + R_C} \right] \\
\frac{v_t}{i_t} &= r_{\pi 2} \parallel R_C \parallel \left[\frac{r_{o2} + R_C}{r_{o2}} \frac{1}{g_{m2} - \frac{1}{R_C}} \right]
\end{aligned}
$$

$$
\begin{aligned}
A_v &= \boxed{-g_{m1} \left(r_{o1} \parallel r_{\pi 2} \parallel R_C \parallel \left[\frac{r_{o2} + R_C}{r_{o2}} \frac{1}{g_{m2} - \frac{1}{R_C}} \right] \right)} \\
R_{in} &= \boxed{r_{\pi 1}} \\
R_{out} &= \boxed{r_{o1} \parallel r_{\pi 2} \parallel R_C \parallel \left[\frac{r_{o2} + R_C}{r_{o2}} \frac{1}{g_{m2} - \frac{1}{R_C}} \right]}
\end{aligned}
$$

(e) From (d), we know the gain from the input to the collector of Q_1 is $-g_{m1} \left(r_{o1} \parallel r_{\pi 2} \parallel R_C \parallel \left[\frac{r_{o2}+R_C}{r_{o2}} \frac{1}{g_{m2}-\frac{1}{R_C}} \right] \right)$. If we find the gain from the collector of Q_1 to v_{out}, we can multiply these expressions to find the overall gain. Let's draw the small-signal model to find the gain from the collector of Q_1 to v_{out}. I'll refer to the collector of Q_1 as node X in the following derivation.



$$\frac{v_{out} - v_X}{R_C} + g_{m2}v_{\pi 2} + \frac{v_{out}}{r_{o2}} = 0$$

$$v_{\pi 2} = v_X$$

$$\frac{v_{out} - v_X}{R_C} + g_{m2}v_X + \frac{v_{out}}{r_{o2}} = 0$$

$$v_{out}\left(\frac{1}{R_C} + \frac{1}{r_{o2}}\right) = v_X\left(\frac{1}{R_C} - g_{m2}\right)$$

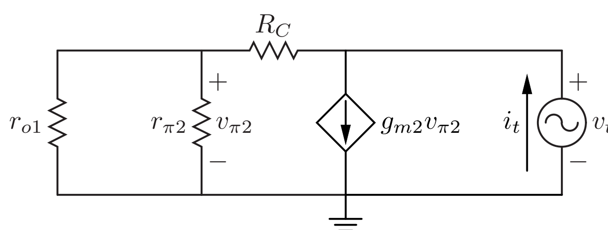
$$\frac{v_{out}}{v_X} = \left(\frac{1}{R_C} - g_{m2}\right)(R_C \parallel r_{o2})$$

Thus, we have

$$A_v = \boxed{-g_{m1}\left(r_{o1} \parallel r_{\pi 2} \parallel R_C \parallel \left[\frac{r_{o2} + R_C}{r_{o2}} \frac{1}{g_{m2} - \frac{1}{R_C}}\right]\right)\left(\frac{1}{R_C} - g_{m2}\right)(R_C \parallel r_{o2})}$$

$$R_{in} = \boxed{r_{\pi 1}}$$

To find the output resistance, let's draw the small-signal model and apply a test source at the output. Note that looking into the collector of Q_1 we see r_{o1}, so we replace Q_1 in the small-signal model with this equivalent resistance. Also note that r_{o2} appears from the output to ground, so we can remove it from this analysis and add it in parallel at the end to find R_{out}.



$$i_t = g_{m2}v_{\pi 2} + \frac{v_{\pi 2}}{r_{\pi 2} \parallel r_{o1}}$$

$$v_{\pi 2} = \frac{r_{\pi 2} \parallel r_{o1}}{r_{\pi 2} \parallel r_{o1} + R_C} v_t$$

$$i_t = \left(g_{m2} + \frac{1}{r_{\pi 2} \parallel r_{o1}}\right) \frac{r_{\pi 2} \parallel r_{o1}}{r_{\pi 2} \parallel r_{o1} + R_C} v_t$$

$$R_{out} = r_{o2} \parallel \frac{v_t}{i_t}$$

$$= \boxed{r_{o2} \parallel \left[\left(\frac{1}{g_{m2}} \parallel r_{\pi 2} \parallel r_{o1}\right) \frac{r_{\pi 2} \parallel r_{o1} + R_C}{r_{\pi 2} \parallel r_{o1}}\right]}$$

40)

Gain of a degenerated CE stage ($V_A = \infty$)

$$A_V = \frac{-R_C}{\frac{1}{g_m} + R_E} = \frac{-R_C g_m}{1 + R_E g_m}$$

$$\frac{\partial A_V}{\partial I_C} = R_C \left(\frac{g_m R_E}{(1 + R_E g_m)^2} \frac{\partial g_m}{\partial I_C} - \frac{\partial g_m / \partial I_C}{1 + g_m R_E} \right)$$

$$\frac{\partial g_m}{\partial I_C} = \frac{1}{V_T} = \frac{1}{26mV} = 38.46 \left(\frac{1}{V}\right)$$

a) $g_m R_E = 3$

$$\frac{\partial A_V}{\partial I_C} = R_C(-2.404) \quad , \quad \partial I_C = 0.1 I_C$$

$$\partial A_V = -R_C I_C (0.24)$$

$$\text{Relative change in gain} = \frac{\partial A_V}{A_V} = \frac{-0.24(R_C I_C)}{-\frac{R_C I_C}{V_T(1 + R_E g_m)}} = 2.5\%$$

40)

b) $g_m R_E = 7$

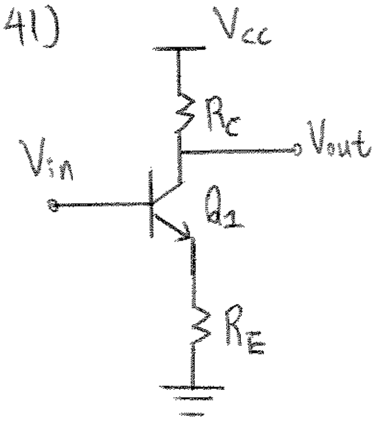
$$\frac{\partial A_v}{\partial I_c} = -R_c\, 0.6$$

$$\partial A_v = -R_c I_c (0.06)$$

Relative change in gain

$$\frac{\partial A_v}{A_v} = \frac{-0.06\,(R_c I_c)}{\dfrac{-R_c I_c}{V_T(1+R_E g_m)}} = 1.25\%$$

41)



$V_A = \infty$

$R_C I_C = 20 V_T$

$R_E I_C = 5 V_T$

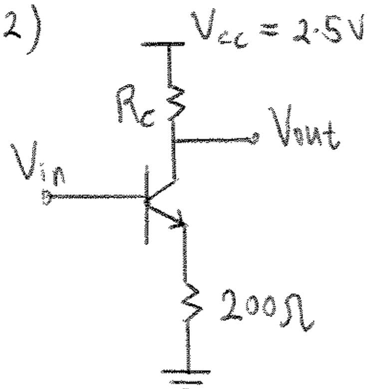
$$|A_v| = \frac{R_C}{R_E + \frac{1}{g_m}} = \frac{R_C}{R_E + \frac{V_T}{I_C}} = \frac{R_C I_C}{R_E I_C + V_T}$$

Assume β is large, so $I_C = I_E$.

$R_C I_C = 20 V_T$, $R_E I_C = 5 V_T$

$$|A_v| = \frac{20 V_T}{5 V_T + V_T} = \frac{20 V_T}{6 V_T} = 3.33$$

42)



$$|A_v| = \frac{R_c I_c}{R_E I_c + V_T} = 10$$

Edge of Saturation

$V_{CE} = V_{BE} = 2.5 - I_c(R_c + R_E)$

$V_{BE} = 0.8V \Rightarrow I_c R_c = 1.7 - I_c 0.2$ (operating point)

$|A_v| = 10 \Rightarrow R_c I_c = 10(R_E I_c + V_T)$ (Gain Equation)

Equating the two equations above $\Rightarrow$

$1.7 - 0.2 I_c = 2 I_c + 0.26 \Rightarrow I_c = 0.655mA$

Check for V_{BE} $\Rightarrow$ $V_{BE} = V_T \ln\left(\frac{I_c}{I_s}\right) = 0.725$, not 0.8, Reiterate

$I_c R_c = 1.775 - I_c 0.2$ (operating point)

$I_c R_c = 2I_c + 0.26$ (Gain equation)

Equating the two equations $\Rightarrow I_c = 0.689mA$

Check for V_{BE} $\Rightarrow$ $V_{BE} = V_T \ln\left(\frac{I_c}{I_s}\right) = 0.727V$, iterate 1 more time

$I_c R_c = 1.773 - I_c 0.2$ (operating point)

$I_c R_c = 2I_c + 0.26$ (Gain equation)

42)

Equating the two equations $\Rightarrow$ $I_c = 0.688\ mA$

Check for V_{BE} $\Rightarrow$ $V_{BE} = V_T \ln\left(\frac{I_c}{I_s}\right) = 0.727V$, converged

$I_c = 0.688mA$

$$R_c = \frac{2I_c + 0.26}{I_c} = \frac{(2)(0.688) + 0.26}{0.688}$$

$R_c = 2.38K\Omega$

$R_{in} = r_\pi + (1+\beta) R_E$

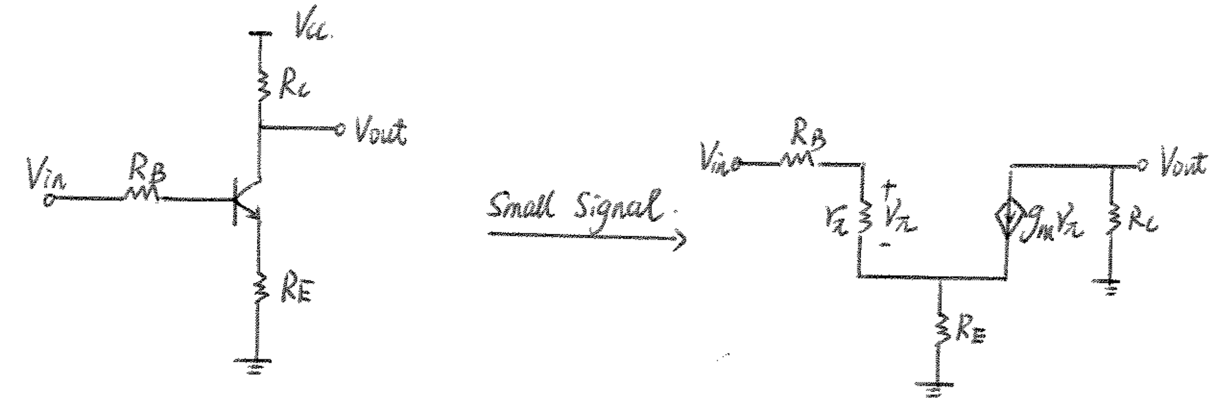
$$R_{in} = \frac{\beta}{g_m} + (101)(0.2) = 24.0K\Omega$$

5.43

$$
\begin{aligned}
A_v &= -\frac{R_C}{\frac{1}{g_m} + (200\ \Omega)} \\
&= -\frac{R_C}{\frac{V_T}{I_C} + (200\ \Omega)} \\
&= -100 \\
R_C &= 100\frac{V_T}{I_C} + 100(200\ \Omega) \\
I_C R_C - I_E(200\ \Omega) &= V_{CE} = V_{BE} = V_T \ln(I_C/I_S) \\
I_C\left(100\frac{V_T}{I_C} + 100(200\ \Omega)\right) - \frac{1+\beta}{\beta} I_C(200\ \Omega) &= V_T \ln(I_C/I_S)
\end{aligned}
$$

We can see that this equation has no solution. For example, if we let $I_C = 0$, we see that according to the left side, we should have $V_{BE} = 2.6$ V, which is clearly an infeasible value. Qualitatively, we know that in order to achieve a large gain, we need a large value for R_C. However, increasing R_C will result in a smaller value of V_{CE}, eventually driving the transistor into saturation. When $A_v = -100$, there is no value of R_C that will provide such a large gain without driving the transistor into saturation.

44) $V_A = \infty$



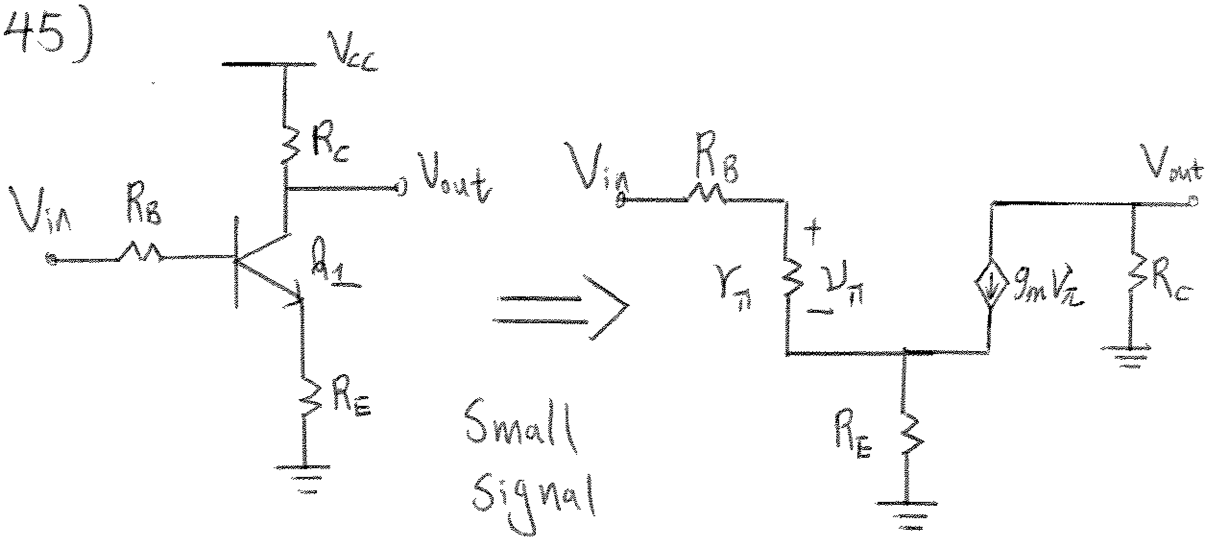
$$V_{out} = -g_m v_\pi R_C$$

$$v_\pi = \frac{V_{in} r_\pi}{R_B + r_\pi + (\beta+1) R_E}$$

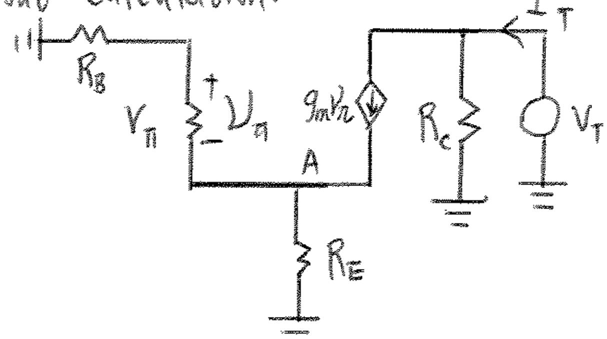
$$V_{out} = \frac{-g_m r_\pi R_C V_{in}}{R_B + r_\pi + (\beta+1) R_E} = \frac{-\beta R_C V_{in}}{R_B + r_\pi + (\beta+1) R_E} = \frac{-R_C V_{in}}{\frac{R_B}{\beta} + \frac{1}{g_m} + \frac{\beta+1}{\beta} R_E}$$

$$\frac{V_{out}}{V_{in}} \approx \frac{-R_C}{\frac{R_B}{\beta+1} + \frac{1}{g_m} + R_E}$$

45)



Rout Calculation:



$V_A = g_m v_\pi \,(R_E \,//\, R_B + r_\pi)$ (1)

$v_\pi = -\dfrac{V_A r_\pi}{r_\pi + R_B} \Rightarrow V_A = -\dfrac{v_\pi (r_\pi + R_B)}{r_\pi}$ (2)

The only possible solution for 1) and 2) is $v_\pi = V_A = 0$,

since 1) is positive and 2) is negative.

$v_\pi = 0 \Rightarrow g_m v_\pi \Rightarrow 0 \Rightarrow \dfrac{V_T}{I_T} = R_c$

Therefore, $R_{out} = R_c$

5.46 (a)

$$A_v = \boxed{-\frac{R_1 + \frac{1}{g_{m2}} \parallel r_{\pi 2}}{\frac{1}{g_{m1}} + R_E}}$$

$$R_{in} = \boxed{r_{\pi 1} + (1+\beta_1)R_E}$$

$$R_{out} = \boxed{R_1 + \frac{1}{g_{m2}} \parallel r_{\pi 2}}$$

(b)

$$A_v = \boxed{-\frac{R_C}{\frac{1}{g_{m1}} + \frac{1}{g_{m2}} \parallel r_{\pi 2}}}$$

$$R_{in} = \boxed{r_{\pi 1} + (1+\beta_1)\left(\frac{1}{g_{m2}} \parallel r_{\pi 2}\right)}$$

$$R_{out} = \boxed{R_C}$$

(c)

$$A_v = \boxed{-\frac{R_C}{\frac{1}{g_{m1}} + \frac{1}{g_{m2}} \parallel r_{\pi 2}}}$$

$$R_{in} = \boxed{r_{\pi 1} + (1+\beta_1)\left(\frac{1}{g_{m2}} \parallel r_{\pi 2}\right)}$$

$$R_{out} = \boxed{R_C}$$

(d)

$$A_v = \boxed{-\frac{R_C}{\frac{1}{g_{m1}} + \frac{1}{g_{m2}} \parallel r_{\pi 2} + \frac{R_B}{1+\beta_1}}}$$

$$R_{in} = \boxed{R_B + r_{\pi 1} + (1+\beta_1)\left(\frac{1}{g_{m2}} \parallel r_{\pi 2}\right)}$$

$$R_{out} = \boxed{R_C}$$

(e)

$$A_v = \boxed{-\frac{R_C}{\frac{1}{g_{m1}} + \frac{1}{g_{m2}} \parallel r_{\pi 2} + \frac{R_B}{1+\beta_1}}}$$

$$R_{in} = \boxed{R_B + r_{\pi 1} + (1+\beta_1)\left(\frac{1}{g_{m2}} \parallel r_{\pi 2}\right)}$$

$$R_{out} = \boxed{R_C}$$

5.47 (a)

$$A_v = \boxed{-\frac{R_C + \frac{1}{g_{m2}} \parallel r_{\pi 2}}{\frac{1}{g_{m1}} + R_E}}$$

$$R_{in} = \boxed{r_{\pi 1} + (1 + \beta_1) R_E}$$

$$R_{out} = \boxed{R_C + \frac{1}{g_{m2}} \parallel r_{\pi 2}}$$

(b)

$$A_v = -\frac{R_C + \frac{1}{g_{m2}} \parallel r_{\pi 2}}{\frac{1}{g_{m1}} + R_E} \cdot \frac{\frac{1}{g_{m2}} \parallel r_{\pi 2}}{R_C + \frac{1}{g_{m2}} \parallel r_{\pi 2}}$$

$$= \boxed{-\frac{\frac{1}{g_{m2}} \parallel r_{\pi 2}}{\frac{1}{g_{m1}} + R_E}}$$

$$R_{in} = \boxed{r_{\pi 1} + (1 + \beta_1) R_E}$$

$$R_{out} = \boxed{\frac{1}{g_{m2}} \parallel r_{\pi 2}}$$

(c)

$$A_v = \boxed{-\frac{R_C + \frac{1}{g_{m2}} \parallel r_{\pi 2}}{\frac{1}{g_{m1}} + \frac{1}{g_{m3} \parallel r_{\pi 3}}}}$$

$$R_{in} = \boxed{r_{\pi 1} + (1 + \beta_1) \left(\frac{1}{g_{m3} \parallel r_{\pi 3}} \right)}$$

$$R_{out} = \boxed{R_C + \frac{1}{g_{m2}} \parallel r_{\pi 2}}$$

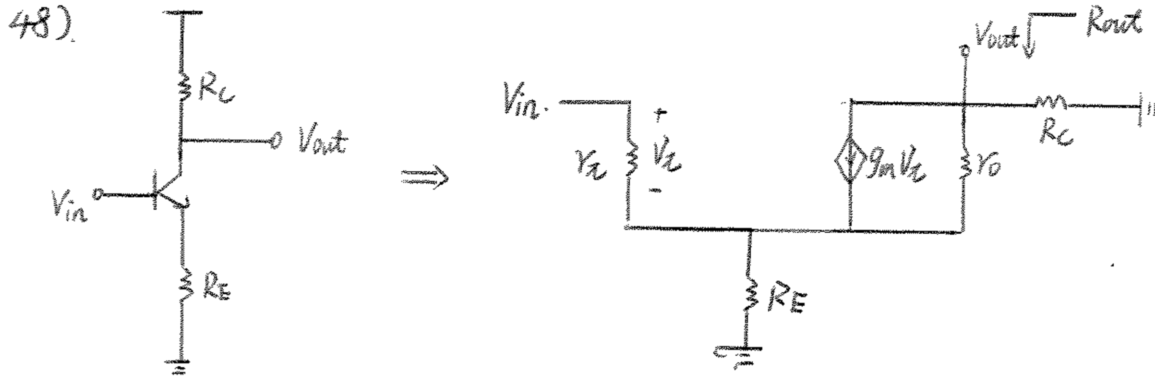
(d)

$$A_v = \boxed{-\frac{R_C \parallel r_{\pi 2}}{\frac{1}{g_{m1}} + R_E}}$$

$$R_{in} = \boxed{r_{\pi 1} + (1 + \beta_1) R_E}$$

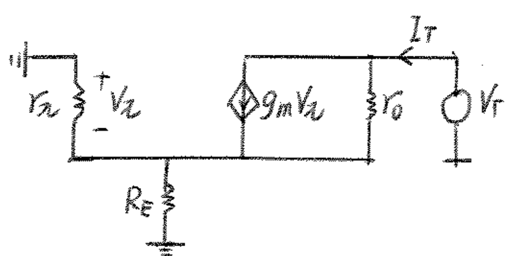
$$R_{out} = \boxed{R_C \parallel r_{\pi 2}}$$

48)



$$R_{out} = R_C \parallel R_{eq}$$

Solve for R_{eq}.



$$I_T = g_m v_\pi + \frac{(V_T + v_\pi)}{r_o}$$

$$v_\pi = -I_T (r_\pi \parallel R_E)$$

$$I_T = -g_m I_T (r_\pi \parallel R_E) + \frac{(V_T - I_T (r_\pi \parallel R_E))}{r_o}$$

$$\frac{V_T}{I_T} = r_o \left(1 + \frac{(r_\pi \parallel R_E)}{r_o} \right) + g_m (r_\pi \parallel R_E))$$

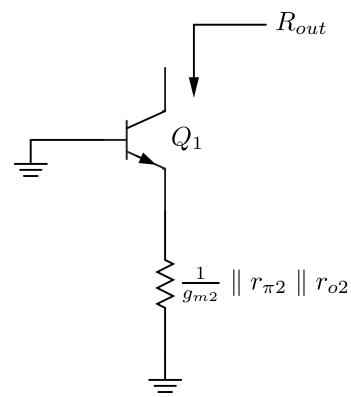
$$\frac{V_T}{I_T} = r_o + (1 + g_m r_o)(r_\pi \parallel R_E)$$

$$R_{eq} = r_o + (1 + g_m r_o)(r_\pi \parallel R_E)$$

$$R_{out} = R_C \parallel r_o + (1 + g_m r_o)(r_\pi \parallel R_E)$$

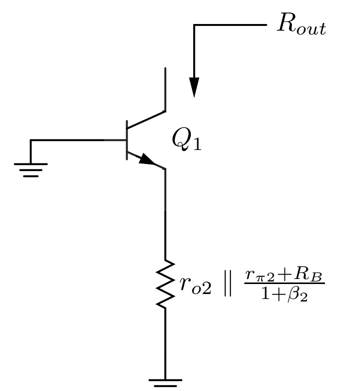
$$R_{out} \approx R_C \parallel r_o (1 + g_m (r_\pi \parallel R_E)) \quad \text{since } g_m r_o \gg 1$$

5.49 (a) Looking into the emitter of Q_2 we see an equivalent resistance of $\frac{1}{g_{m2}} \parallel r_{\pi 2} \parallel r_{o2}$, so we can draw the following equivalent circuit for finding R_{out}:



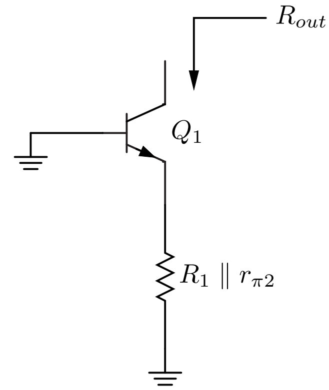
$$R_{out} = \boxed{r_{o1} + (1 + g_{m1} r_{o1}) \left(r_{\pi 1} \parallel \frac{1}{g_{m2}} \parallel r_{\pi 2} \parallel r_{o2} \right)}$$

(b) Looking into the emitter of Q_2 we see an equivalent resistance of $r_{o2} \parallel \frac{r_{\pi 2} + R_B}{1+\beta_2}$ (r_{o2} simply appears in parallel with the resistance seen when $V_A = \infty$), so we can draw the following equivalent circuit for finding R_{out}:



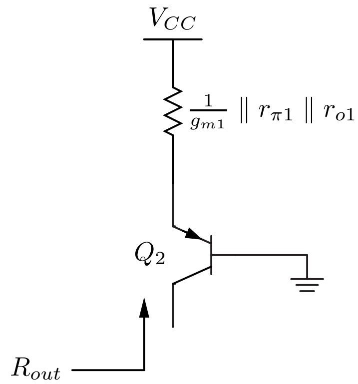
$$R_{out} = \boxed{r_{o1} + (1 + g_{m1} r_{o1}) \left(r_{\pi 1} \parallel r_{o2} \parallel \frac{r_{\pi 2} + R_B}{1 + \beta_2} \right)}$$

(c) Looking down from the emitter of Q_1 we see an equivalent resistance of $R_1 \parallel r_{\pi 2}$, so we can draw the following equivalent circuit for finding R_{out}:



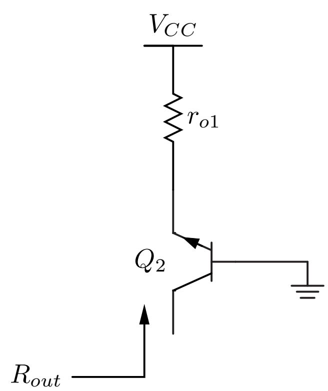
$$R_{out} = \boxed{r_{o1} + (1 + g_{m1} r_{o1}) (r_{\pi 1} \parallel R_1 \parallel r_{\pi 2})}$$

5.50 (a) Looking into the emitter of Q_1 we see an equivalent resistance of $\frac{1}{g_{m1}} \| r_{\pi 1} \| r_{o1}$, so we can draw the following equivalent circuit for finding R_{out}:



$$R_{out} = \boxed{r_{o2} + (1 + g_{m2} r_{o2}) \left(r_{\pi 2} \| \frac{1}{g_{m1}} \| r_{\pi 1} \| r_{o1} \right)}$$

(b) Looking into the emitter of Q_1 we see an equivalent resistance of r_{o1}, so we can draw the following equivalent circuit for finding R_{out}:



$$R_{out} = \boxed{r_{o2} + (1 + g_{m2} r_{o2}) (r_{\pi 2} \| r_{o1})}$$

Comparing this to the solution to part (a), we can see that the output resistance is larger because instead of a factor of $1/g_{m1}$ dominating the parallel resistors in the expression, $r_{\pi 2}$ dominates (assuming $r_{o1} \gg r_{\pi 2}$).

51). $r_\pi = \beta V_T / I_C$.

$$R_{in} = r_\pi // R_B = \frac{\frac{\beta V_T}{I_C} R_B}{\frac{\beta V_T}{I_C} + R_B} = \frac{V_T R_B}{V_T + \frac{I_C}{\beta} R_B} = \frac{V_T R_B}{V_T + I_B R_B}$$

Since $I_B R_B >> V_T \Rightarrow R_{in} \approx \frac{V_T R_B}{I_B R_B} = \frac{V_T}{I_B} = \frac{V_T}{\frac{I_C}{\beta}} = \frac{\beta V_T}{I_C} \approx r_\pi$

So $R_{in} = r_\pi // R_B \approx r_\pi$.

5.52 (a)

$$V_{CC} - I_B(100\text{ k}\Omega) - I_E(100\ \Omega) = V_{BE} = V_T \ln(I_C/I_S)$$
$$V_{CC} - \frac{1}{\beta}I_C(100\text{ k}\Omega) - \frac{1+\beta}{\beta}I_C(100\ \Omega) = V_T \ln(I_C/I_S)$$
$$I_C = 1.6\text{ mA}$$
$$A_v = -\frac{1\text{ k}\Omega}{\frac{1}{g_m} + 100\ \Omega}$$
$$g_m = 61.6\text{ mS}$$
$$A_v = \boxed{-8.60}$$

(b)

$$V_{CC} - I_B(50\text{ k}\Omega) - I_E(2\text{ k}\Omega) = V_T \ln(I_C/I_S)$$
$$I_C = 708\ \mu\text{A}$$
$$A_v = -\frac{1\text{ k}\Omega}{\frac{1}{g_m} + \frac{(1\text{ k}\Omega)\|(50\text{ k}\Omega)}{1+\beta}}$$
$$g_m = 27.2\text{ mS}$$
$$A_v = \boxed{-21.54}$$

(c)

$$I_B = \frac{I_C}{\beta} = \frac{V_{CC} - V_{BE} - I_E(2.5\text{ k}\Omega)}{14\text{ k}\Omega} - \frac{V_{BE} + I_E(2.5\text{ k}\Omega)}{11\text{ k}\Omega}$$
$$I_C = \beta\frac{V_{CC} - V_T\ln(I_C/I_S) - \frac{1+\beta}{\beta}I_C(2.5\text{ k}\Omega)}{14\text{ k}\Omega} - \beta\frac{V_T\ln(I_C/I_S) + \frac{1+\beta}{\beta}I_C(2.5\text{ k}\Omega)}{11\text{ k}\Omega}$$
$$I_C = 163\ \mu\text{A}$$
$$A_v = -\frac{10\text{ k}\Omega}{\frac{1}{g_m} + 500\ \Omega + \frac{(1\text{ k}\Omega)\|(14\text{ k}\Omega)\|(11\text{ k}\Omega)}{1+\beta}}$$
$$g_m = 6.29\text{ mS}$$
$$A_v = \boxed{-14.98}$$

5.53 (a)

$$
\begin{aligned}
I_C &= \frac{V_{CC} - 1.5\text{ V}}{R_C} \\
&= 4\text{ mA} \\
V_{BE} &= V_T \ln(I_C/I_S) = 832\text{ mV} \\
I_B &= \frac{V_{CC} - V_{BE}}{R_B} = 66.7\ \mu\text{A} \\
\beta &= \frac{I_C}{I_B} = \boxed{60}
\end{aligned}
$$

(b) Assuming the speaker has an impedance of 8 Ω, the gain of the amplifier is

$$
\begin{aligned}
A_v &= -g_m \left(R_C \parallel 8\ \Omega\right) \\
&= -\frac{I_C}{V_T} \left(R_C \parallel 8\ \Omega\right) \\
&= \boxed{-1.19}
\end{aligned}
$$

Thus, the circuit provides greater than unity gain.

5.54 (a)

$$A_v = g_m R_C$$

$$g_m = \frac{I_C}{V_T} = 76.9 \text{ mS}$$

$$A_v = \boxed{38.46}$$

$$R_{in} = \frac{1}{g_m} \parallel r_\pi$$

$$r_\pi = \frac{\beta}{g_m} = 1.3 \text{ k}\Omega$$

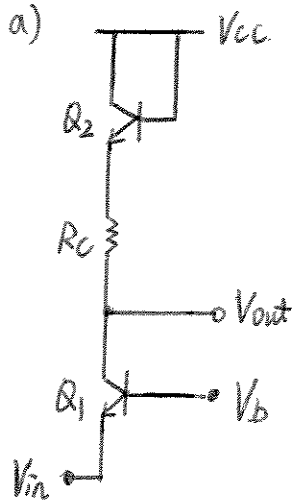
$$R_{in} = \boxed{12.87\ \Omega}$$

$$R_{out} = R_C = \boxed{500\ \Omega}$$

(b) Since $A_v = g_m R_C$ and g_m is fixed for a given value of I_C, R_C should be chosen as large as possible to maximize the gain of the amplifier. V_b should be chosen as small as possible to maximize the headroom of the amplifier (since in order for Q_1 to remain in forward active, we require $V_b < V_{CC} - I_C R_C$).

55) $V_A = \infty$

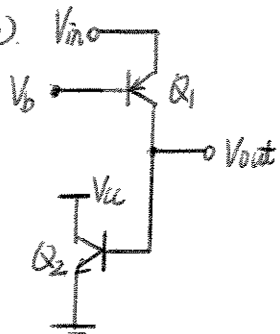
a)



$$|A_v| = \frac{R_C + \frac{1}{g_{m2}} \parallel r_{\pi 2}}{\frac{1}{g_{m1}}}$$

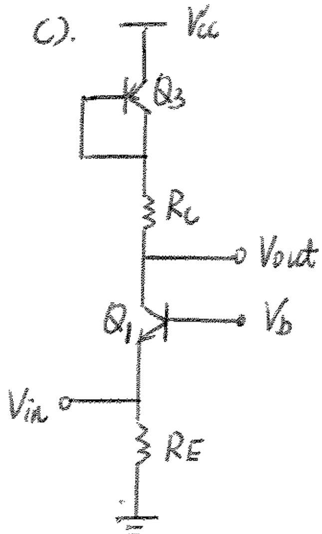
$$= g_{m1}\left(R_C + \frac{1}{g_{m2}} \parallel r_{\pi 2}\right)$$

b).



$$|A_v| = \frac{r_{\pi 2}}{\frac{1}{g_{m1}}} = g_{m1} r_{\pi 2}$$

c).

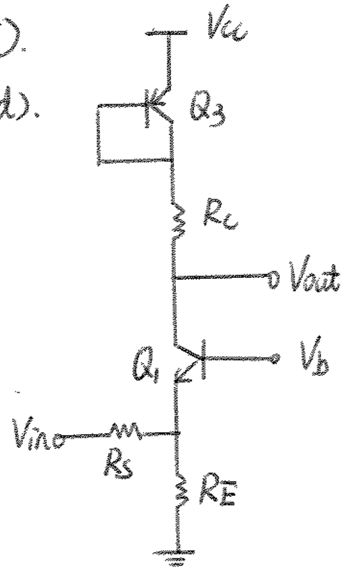


$$|A_v| = \frac{R_C + \frac{1}{g_{m3}} \parallel r_{\pi 3}}{\frac{1}{g_{m1}}}$$

$$= g_{m1}\left(R_C + \frac{1}{g_{m3}} \parallel r_{\pi 3}\right)$$

55).

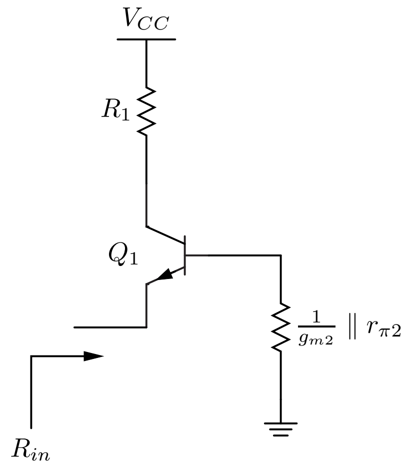
d).



$$|A_v| = \left|\frac{V_{out}}{V_A}\right| \left|\frac{V_A}{V_{in}}\right|$$

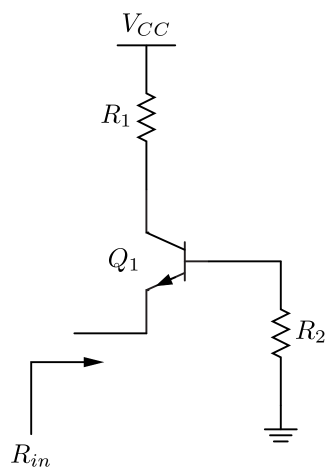
$$= \left[g_{m1} \left(R_C + \frac{1}{g_{m3}} // r_{\pi 3} \right) \right] \left(\frac{R_E // \frac{1}{g_{m1}}}{R_E // \frac{1}{g_{m1}} + R_S} \right)$$

5.56 (a) Looking into the emitter of Q_2 we see an equivalent resistance of $\frac{1}{g_{m2}} \parallel r_{\pi 2}$, so we can draw the following equivalent circuit for finding R_{in}:



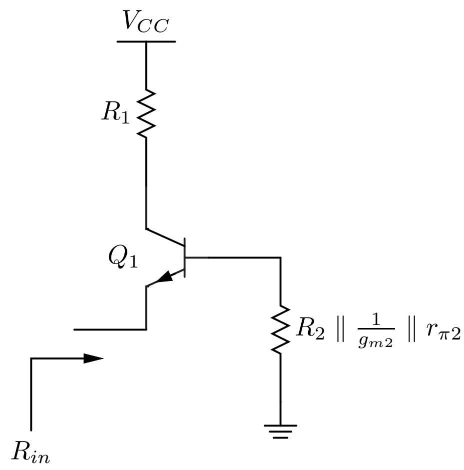
$$R_{in} = \boxed{\frac{r_{\pi 1} + \frac{1}{g_{m2}} \parallel r_{\pi 2}}{1 + \beta_1}}$$

(b) Looking right from the base of Q_1 we see an equivalent resistance of R_2, so we can draw the following equivalent circuit for finding R_{in}:



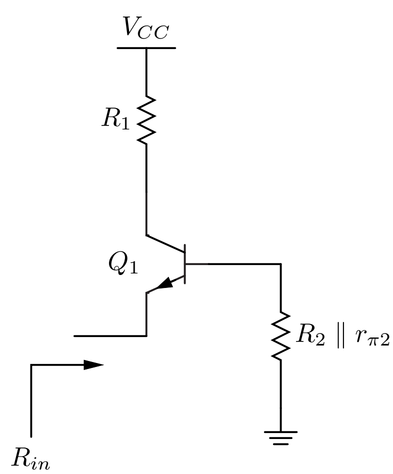
$$R_{in} = \boxed{\frac{r_{\pi 1} + R_2}{1 + \beta_1}}$$

(c) Looking right from the base of Q_1 we see an equivalent resistance of $R_2 \parallel \frac{1}{g_{m2}} \parallel r_{\pi 2}$, so we can draw the following equivalent circuit for finding R_{in}:



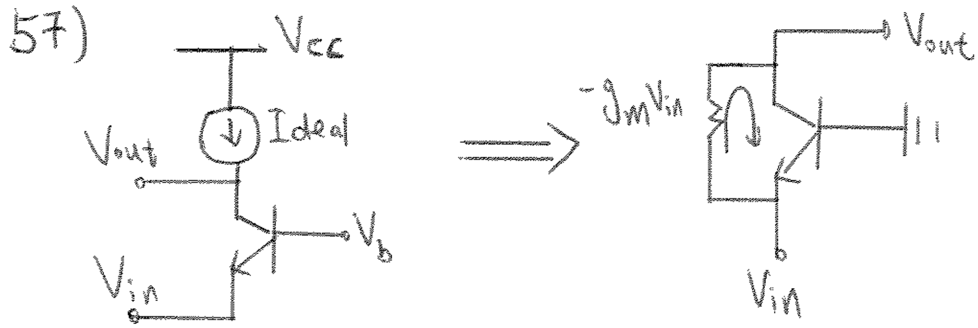
$$R_{in} = \boxed{\frac{r_{\pi 1} + R_2 \parallel \frac{1}{g_{m2}} \parallel r_{\pi 2}}{1 + \beta_1}}$$

(d) Looking right from the base of Q_1 we see an equivalent resistance of $R_2 \parallel r_{\pi 2}$, so we can draw the following equivalent circuit for finding R_{in}:



$$R_{in} = \boxed{\frac{r_{\pi 1} + R_2 \parallel r_{\pi 2}}{1 + \beta_1}}$$

57)



Since an ideal current source is an open circuit, the signal current produced by the transistor has no where to go but r_o.

So $V_{out} = -(g_m(0 - V_{in}))r_o + V_{in}$

$V_{out} = g_m r_o V_{in} + V_{in}$

$V_{out} = V_{in}(g_m r_o + 1)$

$$\frac{V_{out}}{V_{in}} = 1 + g_m r_o$$

5.58 (a)

$$I_B = \frac{I_C}{\beta} = \frac{V_{CC} - V_{BE} - I_E(400\ \Omega)}{13\ \text{k}\Omega} - \frac{V_{BE} + I_E(400\ \Omega)}{12\ \text{k}\Omega}$$

$$I_C = \beta\frac{V_{CC} - V_T\ln(I_C/I_S) - \frac{1+\beta}{\beta}I_C(400\ \Omega)}{13\ \text{k}\Omega} - \beta\frac{V_T\ln(I_C/I_S) + \frac{1+\beta}{\beta}I_C(400\ \Omega)}{12\ \text{k}\Omega}$$

$$I_C = \boxed{1.02\ \text{mA}}$$

$$V_{BE} = V_T\ln(I_C/I_S) = \boxed{725\ \text{mV}}$$

$$V_{CE} = V_{CC} - I_C(1\ \text{k}\Omega) - I_E(400\ \Omega) = \boxed{1.07\ \text{V}}$$

Q_1 is operating in forward active.

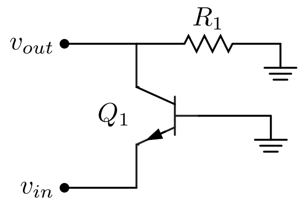
(b)

$$A_v = g_m(1\ \text{k}\Omega)$$

$$g_m = 39.2\ \text{mS}$$

$$A_v = \boxed{39.2}$$

5.61 For small-signal analysis, we can draw the following equivalent circuit.



$$A_v = \boxed{g_m R_1}$$

$$R_{in} = \boxed{\frac{1}{g_m} \parallel r_\pi}$$

$$R_{out} = \boxed{R_1}$$

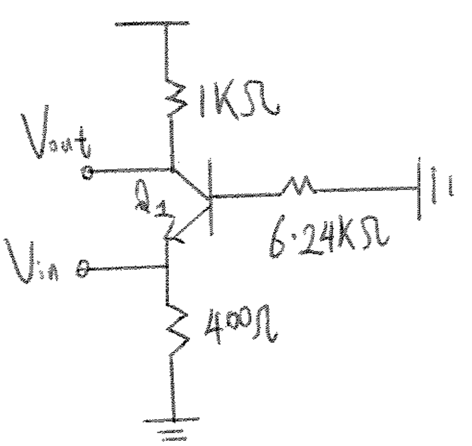
59)

$C_B = 0$

a) Since C_B was not considered during DC analysis, it has no effect on operating point analysis. So it is still the same as 58).

$$V_{BE} = 0.725\,V$$
$$I_c = 1.0163\,mA$$
$$I_B = 10.163\,\mu A$$
$$V_{CE} = 1.07\,V$$

b) Since capacitor is frequency dependent, the circuit's AC analysis will be different.



$$|A_v| = \frac{1k}{\frac{1}{g_m} + \frac{6.24k\Omega}{\beta+1}} = 11.4$$

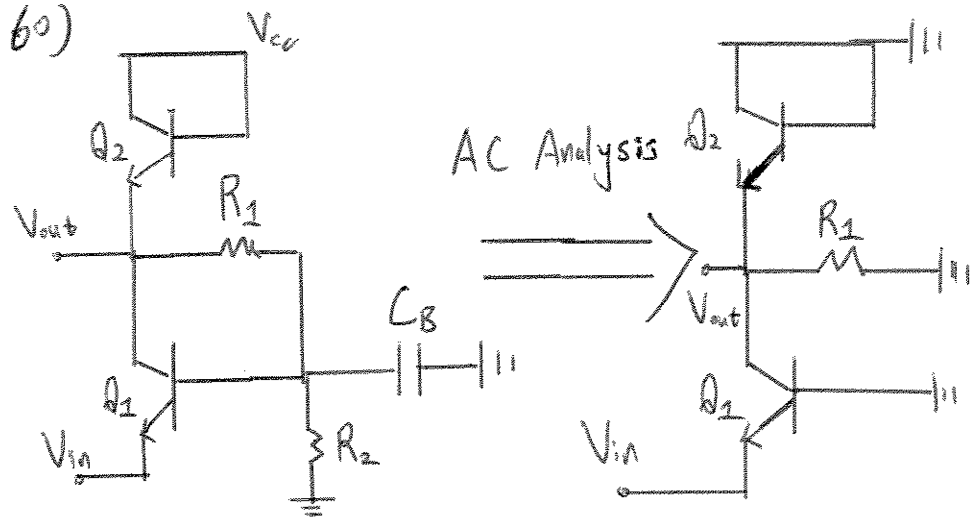
$$R_{in} = 400\Omega \,//\, \left(\frac{1}{g_m} + \frac{6.24k\Omega}{\beta+1}\right)$$

$$R_{in} = 71.7\Omega$$

Note: $6.24\,k\Omega$ is R_{THEV} of $13\,k\Omega$ and $12\,k\Omega$ combination.

$$R_{out} = 1k\Omega$$

60)

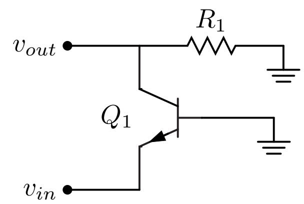


$$R_{out} = \frac{1}{g_{m2}} // r_{\pi 2} // R_1 \approx \frac{1}{g_{m2}} // R_1$$

$$A_v = \left|\frac{V_{out}}{V_{in}}\right| = g_{m1}\left(\frac{1}{g_{m2}} // r_{\pi 2} // R_1\right) \approx g_{m1}\left(\frac{1}{g_{m2}} // R_1\right)$$

$$R_{in} = \frac{1}{g_m} // r_{\pi 1} \approx \frac{1}{g_{m1}}$$

5.61 For small-signal analysis, we can draw the following equivalent circuit.

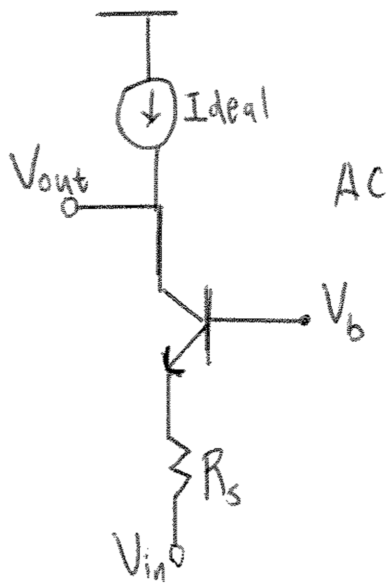


$$A_v = \boxed{g_m R_1}$$

$$R_{in} = \boxed{\frac{1}{g_m} \parallel r_\pi}$$

$$R_{out} = \boxed{R_1}$$

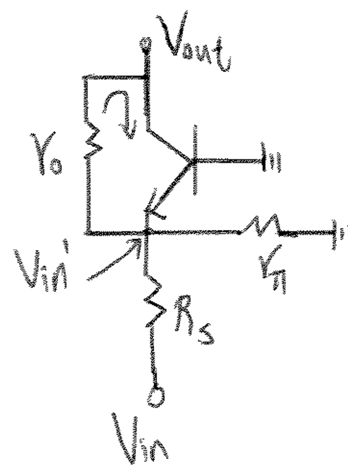
62)



$V_A < \infty$

AC Analysis

$\Longrightarrow$



$$A_v = \frac{V_{out}}{V_{in}} = \left(\frac{V_{in}'}{V_{in}}\right)\left(\frac{V_{out}}{V_{in}'}\right), \quad \left(\frac{V_{in}'}{V_{in}}\right) = \frac{r_\pi}{r_\pi + R_s}$$

Since V_{out} is float, so looking at emitter and r_o, we will see an infinite impedance.

$$\frac{V_{out}}{V_{in}'} \Rightarrow -g_m(-V_{in}')r_o + V_{in}' = V_{out} \Rightarrow \frac{V_{out}}{V_{in}'} = (g_m r_o + 1)$$

$$A_v = (g_m r_o + 1)\left(\frac{r_\pi}{r_\pi + R_s}\right).$$

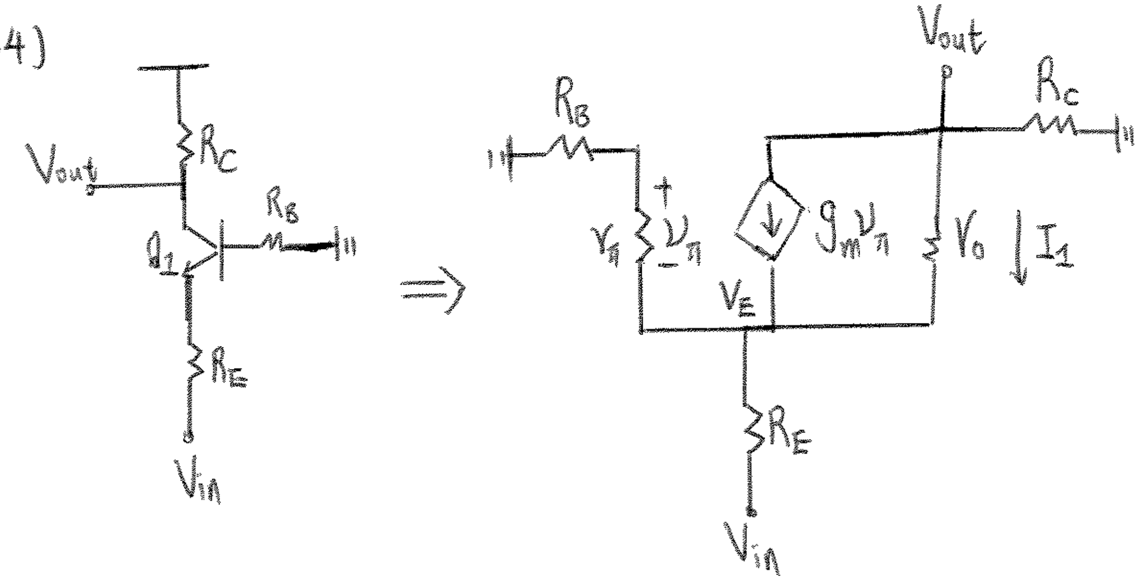
5.63 Since $I_{S1} = 2I_{S2}$ and they're biased identically, we know that $I_{C1} = 2I_{C2}$, which means $g_{m1} = 2g_{m2}$.

$$\frac{v_{out1}}{v_{in}} = g_{m1}R_C = 2g_{m2}R_C$$

$$\frac{v_{out2}}{v_{in}} = g_{m2}R_C$$

$$\Rightarrow \boxed{\frac{v_{out1}}{v_{in}} = 2\frac{v_{out2}}{v_{in}}}$$

64)



$$V_{out} = -(I_1 + g_m v_\pi) R_c \,, \quad I_1 = \frac{V_{out} - V_E}{r_o}$$

$$V_{out} = -\left(\frac{V_{out} - V_E}{r_o} + g_m v_\pi\right) R_c \,, \quad V_E = \frac{-g_m v_\pi}{\beta}(r_\pi + R_B)$$

$$V_{out} = -\left(\frac{V_{out} + \frac{g_m v_\pi}{\beta}(r_\pi + R_B)}{r_o} + g_m v_\pi\right) R_c$$

Rearranging

$$v_\pi = \frac{-\left(1 + \frac{R_c}{r_o}\right)}{\frac{g_m (r_\pi + R_B) R_c}{\beta r_o} + g_m R_c} V_{out} = A V_{out}$$

Summing the voltage at node E.

$$V_E - \left(\left(1 + \frac{1}{\beta}\right) g_m v_\pi + \frac{(V_{out} - V_E)}{r_o}\right) R_E = V_{in} \qquad (1)$$

64)

Writing V_E in terms of V_π, and v_π in terms of V_{out}

1) becomes

$$-\frac{g_m A V_{out}}{\beta}(r_\pi + R_B)\left(1+\frac{R_E}{r_o}\right) - \left(1+\frac{1}{\beta}\right) g_m A V_{out} R_E - \frac{V_{out}}{r_o} R_E = V_{in}$$

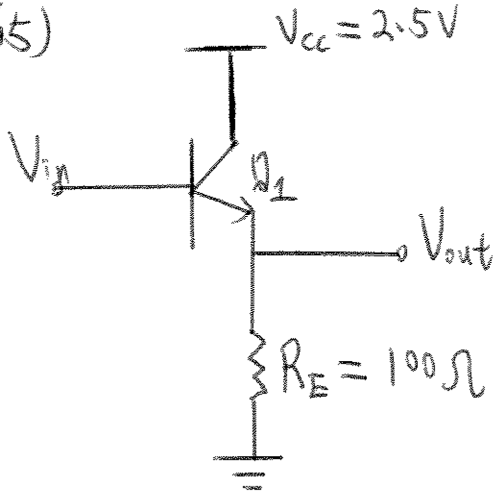
Solving V_{out}/V_{in} =>

$$\frac{V_{out}}{V_{in}} = \frac{1}{-\frac{g_m A}{\beta}(r_\pi + R_B)\left(1+\frac{R_E}{r_o}\right) - \left(1+\frac{1}{\beta}\right) g_m A R_E - \frac{R_E}{r_o}}$$

subsituting A into equation

$$\frac{V_{out}}{V_{in}} = \frac{\frac{g_m (r_\pi + R_B) R_c}{\beta r_o} + g_m R_c}{g_m\left(1+\frac{R_c}{r_o}\right)(r_\pi + R_B)\left(1+\frac{R_E}{r_o}\right) + \left(1+\frac{1}{\beta}\right) g_m \left(1+\frac{R_c}{r_o}\right) R_E - \frac{R_E}{r_o}\left(\frac{g_m (r_\pi + R_B) R_c}{\beta r_o} + g_m R_c\right)}$$

65)



$R_E = 100\Omega$
$V_A = \infty$
$|A_v| = 0.8$

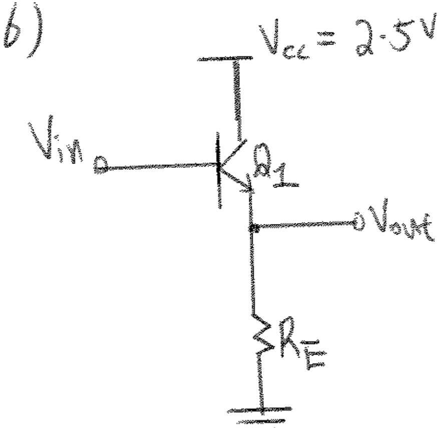
$$|A_v| = \frac{R_E}{R_E + \frac{1}{g_m}} = \frac{R_E I_c}{R_E I_c + V_T} = 0.8$$

$\Rightarrow R_E I_c = 0.8(R_E I_c + V_T)$, $R_E = 100\Omega$

$\Rightarrow 0.1 I_c = 0.08 I_c + 0.0208 \Rightarrow 0.02 I_c = 0.0208$

$\Rightarrow I_c = 1.04mA$

6b)



$|A_v| > 0.9$
$R_{in} > 10K\Omega$

$$|A_v| = \frac{R_E I_c}{R_E I_c + V_T} > 0.9 \Rightarrow R_E I_c > 0.9[R_E I_c + V_T]$$

$$\Rightarrow R_E I_c > 9V_T = 234\,mV, \text{ Let } R_E I_c = 240\,mV$$

$$R_{in} = r_\pi + (1+\beta)R_E > 10K \Rightarrow 100V_T + (101)R_E I_c > 10K\Omega I_c$$

Subsituting $R_E I_c = 240\,mV \Rightarrow I_c < 2.684\,mA$

Choose I_c to be $2.5\,mA \Rightarrow R_E = 96\Omega$

To Verify:

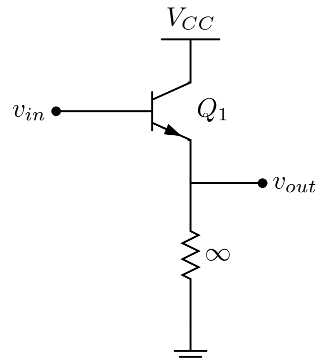
$$R_{in} = 100\frac{(0.026)}{2.5} + (101)0.096 = 10.74K\Omega$$

$$|A_v| = \frac{(0.096)(2.5)}{(0.096)(2.5) + 0.026} = 0.902$$

5.67

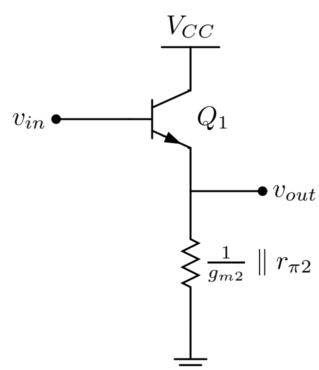
$$
\begin{aligned}
R_{out} &= \frac{r_\pi + R_S}{1+\beta} \\
&= \frac{\beta V_T / I_C + R_S}{1+\beta} \\
&\leq 5\ \Omega \\
I_C &= \frac{\beta}{1+\beta} I_E = \frac{\beta}{1+\beta} I_1 \\
\frac{\frac{\beta(1+\beta)V_T}{\beta I_1} + R_S}{1+\beta} &= \frac{\frac{(1+\beta)V_T}{I_1} + R_S}{1+\beta} \\
&\leq 5\ \Omega \\
I_1 &\geq \boxed{8.61\ \text{mA}}
\end{aligned}
$$

5.68 (a) Looking into the collector of Q_2 we see an equivalent resistance of $r_{o2} = \infty$, so we can draw the following equivalent circuit:



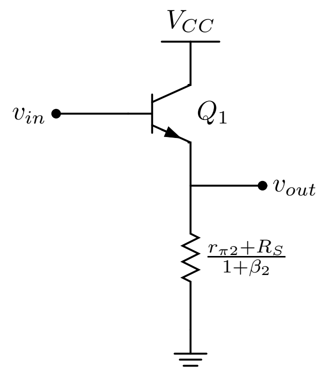
$$A_v = \boxed{1}$$
$$R_{in} = \boxed{\infty}$$
$$R_{out} = \boxed{\frac{1}{g_{m1}} \parallel r_{\pi 1}}$$

(b) Looking down from the emitter of Q_1 we see an equivalent resistance of $\frac{1}{g_{m2}} \parallel r_{\pi 2}$, so we can draw the following equivalent circuit:



$$A_v = \boxed{\frac{\frac{1}{g_{m2}} \parallel r_{\pi 2}}{\frac{1}{g_{m1}} + \frac{1}{g_{m2}} \parallel r_{\pi 2}}}$$
$$R_{in} = \boxed{r_{\pi 1} + (1 + \beta_1)\left(\frac{1}{g_{m2}} \parallel r_{\pi 2}\right)}$$
$$R_{out} = \boxed{\frac{1}{g_{m1}} \parallel r_{\pi 1} \parallel \frac{1}{g_{m2}} \parallel r_{\pi 2}}$$

(c) Looking into the emitter of Q_2 we see an equivalent resistance of $\frac{r_{\pi 2} + R_S}{1 + \beta_2}$, so we can draw the following equivalent circuit:

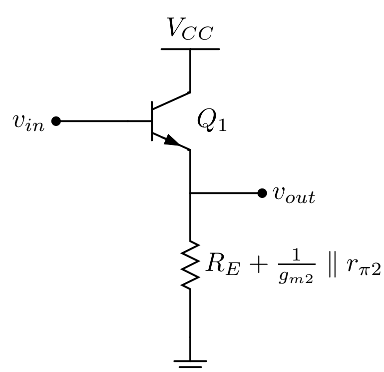


$$A_v = \boxed{\frac{\frac{r_{\pi 2}+R_S}{1+\beta_2}}{\frac{1}{g_{m1}} + \frac{r_{\pi 2}+R_S}{1+\beta_2}}}$$

$$R_{in} = \boxed{r_{\pi 1} + (1+\beta_1)\left(\frac{r_{\pi 2}+R_S}{1+\beta_2}\right)}$$

$$R_{out} = \boxed{\frac{1}{g_{m1}} \parallel r_{\pi 1} \parallel \left(\frac{r_{\pi 2}+R_S}{1+\beta_2}\right)}$$

(d) Looking down from the emitter of Q_1 we see an equivalent resistance of $R_E + \frac{1}{g_{m2}} \parallel r_{\pi 2}$, so we can draw the following equivalent circuit:

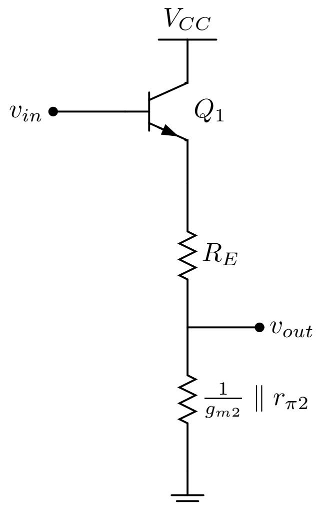


$$A_v = \boxed{\frac{R_E + \frac{1}{g_{m2}} \parallel r_{\pi 2}}{\frac{1}{g_{m1}} + R_E + \frac{1}{g_{m2}} \parallel r_{\pi 2}}}$$

$$R_{in} = \boxed{r_{\pi 1} + (1+\beta_1)\left(R_E + \frac{1}{g_{m2}}\right)}$$

$$R_{out} = \boxed{\frac{1}{g_{m1}} \parallel r_{\pi 1} \parallel \left(R_E + \frac{1}{g_{m2}}\right)}$$

(e) Looking into the emitter of Q_2 we see an equivalent resistance of $\frac{1}{g_{m2}} \parallel r_{\pi 2}$, so we can draw the following equivalent circuit:



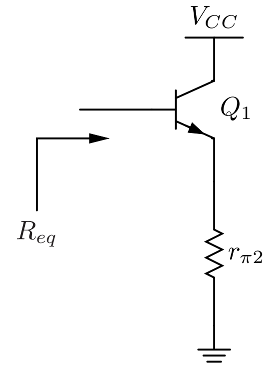
$$A_v = \frac{R_E + \frac{1}{g_{m2}} \parallel r_{\pi 2}}{\frac{1}{g_{m1}} + R_E + \frac{1}{g_{m2}} \parallel r_{\pi 2}} \cdot \frac{\frac{1}{g_{m2}} \parallel r_{\pi 2}}{R_E + \frac{1}{g_{m2}} \parallel r_{\pi 2}}$$

$$= \boxed{\frac{\frac{1}{g_{m2}} \parallel r_{\pi 2}}{\frac{1}{g_{m1}} + R_E + \frac{1}{g_{m2}} \parallel r_{\pi 2}}}$$

$$R_{in} = \boxed{r_{\pi 1} + (1 + \beta_1)\left(R_E + \frac{1}{g_{m2} \parallel r_{\pi 2}}\right)}$$

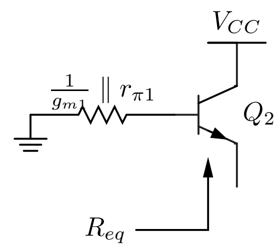
$$R_{out} = \boxed{\left(\frac{1}{g_{m1}} \parallel r_{\pi 1} + R_E\right) \parallel \frac{1}{g_{m2}} \parallel r_{\pi 2}}$$

5.69 (a) Looking into the base of Q_2 we see an equivalent resistance of $r_{\pi 2}$ (assuming the emitter of Q_2 is grounded), so we can draw the following equivalent circuit for finding the impedance at the base of Q_1:



$$R_{eq} = \boxed{r_{\pi 1} + (1 + \beta_1) r_{\pi 2}}$$

(b) Looking into the emitter of Q_1 we see an equivalent resistance of $\frac{1}{g_{m1}} \parallel r_{\pi 1}$ (assuming the base of Q_1 is grounded), so we can draw the following equivalent circuit for finding the impedance at the emitter of Q_2:



$$R_{eq} = \boxed{\frac{r_{\pi 2} + \frac{1}{g_{m1}} \parallel r_{\pi 1}}{1 + \beta_2}}$$

(c)

$$\frac{I_{C1} + I_{C2}}{I_{B1}} = \frac{\beta_1 I_{B1} + \beta_2 (1 + \beta_1) I_{B1}}{I_{B1}}$$
$$= \boxed{\beta_1 + \beta_2 (1 + \beta_1)}$$

If we assume that $\beta_1, \beta_2 \gg 1$, then this simplifies to $\beta_1 \beta_2$, meaning a Darlington pair has a current gain approximately equal to the product of the current gains of the individual transistors.

5.70 (a)

$$R_{CS} = \boxed{r_{o2} + (1 + g_{m2} r_{o2}) (r_{\pi 2} \parallel R_E)}$$

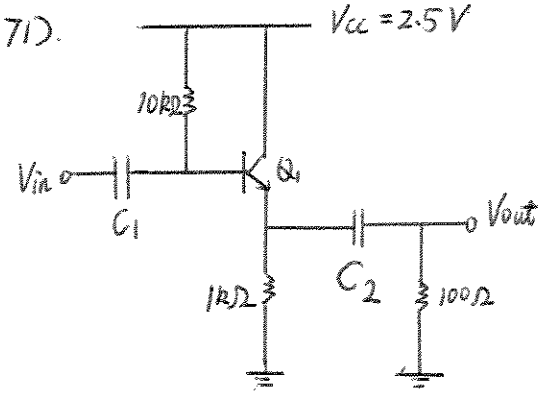
(b)

$$A_v = \boxed{\frac{r_{o2} + (1 + g_{m2} r_{o2}) (r_{\pi 2} \parallel R_E)}{\frac{1}{g_{m1}} + r_{o2} + (1 + g_{m2} r_{o2}) (r_{\pi 2} \parallel R_E)}}$$

$$R_{in} = \boxed{r_{\pi 1} + (1 + \beta_1) [r_{o2} + (1 + g_{m2} r_{o2}) (r_{\pi 2} \parallel R_E)]}$$

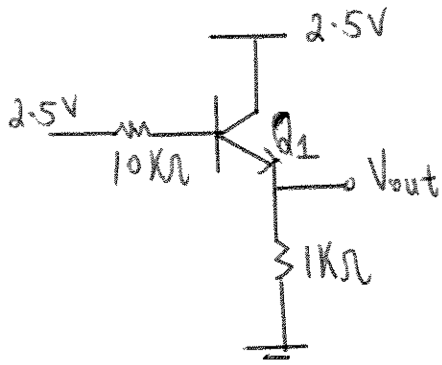
$$R_{out} = \boxed{\frac{1}{g_{m1}} \parallel r_{\pi 1} \parallel [r_{o2} + (1 + g_{m2} r_{o2}) (r_{\pi 2} \parallel R_E)]}$$

71).



$$I_s = 7 \times 10^{-16} A$$
$$\beta = 100$$
$$V_A = 5V$$

DC Analysis: (Ignore r_o's effect)



$$I_c = \beta \left(\frac{2.5 - (V_{BE} + \frac{I_c}{\alpha} 1K\Omega)}{10K\Omega} \right)$$

rearrange

$$I_c = \frac{2.5 - V_{BE}}{\frac{10K\Omega}{\beta} + \frac{1K\Omega}{\alpha}}$$

Guess: $V_{BE} = 0.7V$, $I_c = 1.621 mA$

check for V_{BE}: $V_{BE} = V_T \ln\left(\frac{I_c}{I_s}\right) = 0.740V$, not 0.7, reiterate

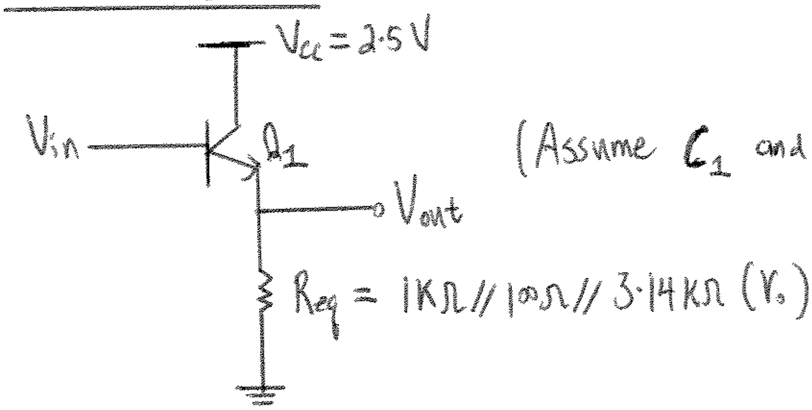
$V_{BE} = 0.740V$, $I_c = 1.59 mA$

Check for V_{BE}: $V_{BE} = V_T \ln\left(\frac{I_c}{I_s}\right) = 0.740V$, Converged.

So $I_c = 1.59mA$, $g_m = 0.0612 (\frac{1}{\Omega}) S$, $\frac{1}{g_m} = 16.34\Omega$,
$r_o = 3.14K\Omega$

71)

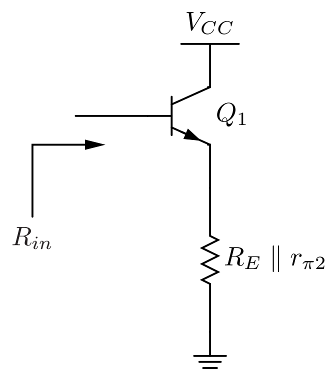
AC Analysis: (Include r_o)



(Assume C_1 and C_2 are large)

$$A_v = \frac{(1K\Omega // 100\Omega // 3.14K\Omega)}{16.34\Omega + (1K\Omega // 100\Omega / 3.14K\Omega)} = 0.84$$

5.72 (a) Looking into the base of Q_2 we see an equivalent resistance of $r_{\pi 2}$, so we can draw the following equivalent circuit for finding R_{in}:

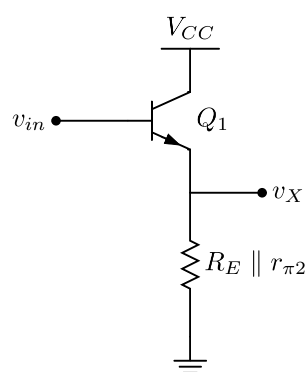


$$R_{in} = \boxed{r_{\pi 1} + (1 + \beta_1)(R_E \parallel r_{o1})}$$

Looking into the collector of Q_2 we see an equivalent resistance of r_{o2}. Thus,

$$R_{out} = \boxed{R_C \parallel r_{o2}}$$

(b) Looking into the base of Q_2 we see an equivalent resistance of $r_{\pi 2}$, so we can draw the following equivalent circuit for finding v_X/v_{in}:

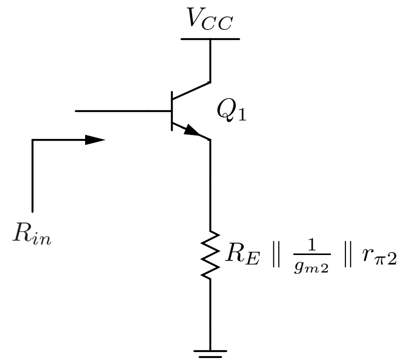


$$\frac{v_X}{v_{in}} = \frac{R_E \parallel r_{\pi 2} \parallel r_{o1}}{\frac{1}{g_{m1}} + R_E \parallel r_{\pi 2} \parallel r_{o1}}$$

We can find v_{out}/v_X by inspection.

$$\begin{aligned}
\frac{v_{out}}{v_X} &= -g_{m2}(R_C \parallel r_{o2}) \\
A_v &= \frac{v_X}{v_{in}} \cdot \frac{v_{out}}{v_X} \\
&= \boxed{-g_{m2}(R_C \parallel r_{o2}) \frac{R_E \parallel r_{\pi 2} \parallel r_{o1}}{\frac{1}{g_{m1}} + R_E \parallel r_{\pi 2} \parallel r_{o1}}}
\end{aligned}$$

5.73 (a) Looking into the emitter of Q_2 we see an equivalent resistance of $\frac{1}{g_{m2}} \parallel r_{\pi 2}$, so we can draw the following equivalent circuit for finding R_{in}:

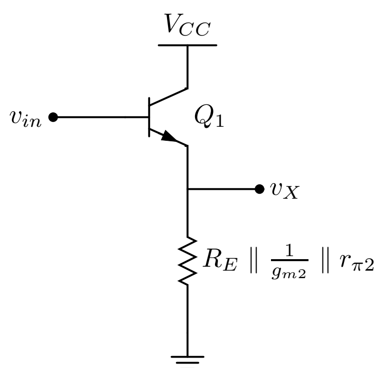


$$R_{in} = \boxed{r_{\pi 1} + (1+\beta_1)\left(R_E \parallel \frac{1}{g_{m2}} \parallel r_{\pi 2}\right)}$$

Looking into the collector of Q_2, we see an equivalent resistance of ∞ (because $V_A = \infty$), so we have

$$R_{out} = \boxed{R_C}$$

(b) Looking into the emitter of Q_2 we see an equivalent resistance of $\frac{1}{g_{m2}} \parallel r_{\pi 2}$, so we can draw the following equivalent circuit for finding v_X/v_{in}:



$$\frac{v_X}{v_{in}} = \frac{R_E \parallel \frac{1}{g_{m2}} \parallel r_{\pi 2}}{\frac{1}{g_{m1}} +_E \parallel \frac{1}{g_{m2}} \parallel r_{\pi 2}}$$

We can find v_{out}/v_X by inspection.

$$\frac{v_{out}}{v_X} = g_{m2} R_C$$

$$A_v = \frac{v_X}{v_{in}} \cdot \frac{v_{out}}{v_X}$$

$$= \boxed{g_{m2} R_C \frac{R_E \parallel \frac{1}{g_{m2}} \parallel r_{\pi 2}}{\frac{1}{g_{m1}} +_E \parallel \frac{1}{g_{m2}} \parallel r_{\pi 2}}}$$

5.74

$$
\begin{aligned}
R_{out} &= R_C = \boxed{1\ \mathrm{k\Omega}} \\
A_v &= -g_m R_C = -10 \\
g_m &= 10\ \mathrm{mS} \\
I_C &= g_m V_T = 260\ \mu\mathrm{A} \\
\frac{V_{CC} - V_{BE}}{R_B} &= I_B = \frac{I_C}{\beta} \\
R_B &= \beta \frac{V_{CC} - V_T \ln(I_C/I_S)}{I_C} \\
&= \boxed{694\ \mathrm{k\Omega}} \\
R_{in} &= R_B \parallel r_\pi = 9.86\ \mathrm{k\Omega} > 5\ \mathrm{k\Omega}
\end{aligned}
$$

In sizing C_B, we must consider the effect a finite impedance in series with the input will have on the circuit parameters. Any series impedance will cause R_{in} to increase and will not impact R_{out}. However, a series impedance can cause gain degradation. Thus, we must ensure that $|Z_B| = \left|\frac{1}{j\omega C_B}\right|$ does not degrade the gain significantly.

If we include $|Z_B|$ in the gain expression, we get:

$$
A_v = -\frac{R_C}{\frac{1}{g_m} + \frac{(|Z_B|)\|R_B}{1+\beta}}
$$

Thus, we want $\frac{1}{1+\beta}|Z_B| \ll \frac{1}{g_m}$ to ensure the gain is not significantly degraded.

$$
\begin{aligned}
\frac{1}{1+\beta}\left|\frac{1}{j\omega C_B}\right| &\ll \frac{1}{g_m} \\
\frac{1}{1+\beta}\frac{1}{2\pi f C_B} &= \frac{1}{10}\frac{1}{g_m} \\
C_B &= \boxed{788\ \mathrm{nF}}
\end{aligned}
$$

5.75

$$R_{out} = R_C \leq 500\ \Omega$$

To maximize gain, we should maximize R_C.

$$R_C = \boxed{500\ \Omega}$$
$$V_{CC} - I_C R_C \geq V_{BE} - 400\text{ mV} = V_T \ln(I_C/I_S) - 400\text{ mV}$$
$$I_C \leq 4.261\text{ mA}$$

To maximize gain, we should maximize I_C.

$$I_C = \boxed{4.261\text{ mA}}$$
$$I_B = \frac{I_C}{\beta} = \frac{V_{CC} - V_{BE}}{R_B}$$
$$= \frac{I_C}{\beta} = \frac{V_{CC} - V_T \ln(I_C/I_S)}{R_B}$$
$$R_B = \boxed{40.613\text{ k}\Omega}$$

5.76

$$
\begin{aligned}
R_{out} &= R_C = \boxed{1\ \text{k}\Omega} \\
|A_v| &= g_m R_C \\
&= \frac{I_C R_C}{V_T} \\
&\geq 20 \\
I_C &\geq 520\ \mu\text{A}
\end{aligned}
$$

In order to maximize $R_{in} = R_B \parallel r_\pi$, we need to maximize r_π, meaning we should minimize I_C (since $r_\pi = \frac{\beta V_T}{I_C}$).

$$
\begin{aligned}
I_C &= 520\ \mu\text{A} \\
I_B &= \frac{I_C}{\beta} = \frac{V_{CC} - V_{BE}}{R_B} \\
&= \frac{V_{CC} - V_T \ln(I_C/I_S)}{R_B} \\
R_B &= \boxed{343\ \text{k}\Omega}
\end{aligned}
$$

5.77

$$\begin{aligned}
R_{out} &= R_C = \boxed{2\text{ k}\Omega} \\
A_v &= -g_m R_C \\
&= -\frac{I_C R_C}{V_T} \\
&= -15 \\
I_C &= 195\ \mu\text{A} \\
V_{BE} &= V_T \ln(I_C/I_S) = 689.2\text{ mV} \\
V_{CE} &\geq V_{BE} - 400\text{ mV} = 289.2\text{ mV}
\end{aligned}$$

To minimize the supply voltage, we should minimize V_{CE}.

$$\begin{aligned}
V_{CE} &= 289.2\text{ mV} \\
\frac{V_{CC} - V_{CE}}{R_C} &= I_C \\
V_{CC} &= 679.2\text{ mV}
\end{aligned}$$

Note that this value of V_{CC} is less than the required V_{BE}. This means that the value of V_{CC} is constrained by V_{BE}, not V_{CE}. In theory, we could pick $V_{CC} = V_{BE}$, but in this case, we'd have to set $R_B = 0\ \Omega$, which would short the input to V_{CC}. Thus, let's pick a reasonable value for R_B, $R_B = \boxed{100\ \Omega}$.

$$\begin{aligned}
\frac{V_{CC} - V_{BE}}{R_B} &= I_B = \frac{I_C}{\beta} \\
V_{CC} &= \boxed{689.4\text{ mV}}
\end{aligned}$$

5.78

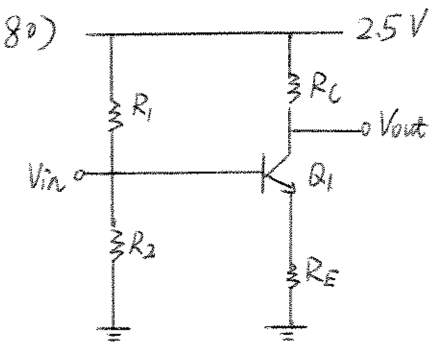
$$
\begin{aligned}
|A_v| &= g_m R_C \\
&= \frac{I_C R_C}{V_T} \\
&= A_0 \\
R_{out} &= R_C \\
A_0 &= \frac{I_C R_{out}}{V_T} \\
I_C &= \frac{A_0 V_T}{R_{out}} \\
P &= I_C V_{CC} \\
&= \boxed{\frac{A_0 V_T}{R_{out}} V_{CC}}
\end{aligned}
$$

Thus, we must trade off a small output resistance with low power consumption (i.e., as we decrease R_{out}, power consumption increases and vice-versa).

5.79

$$
\begin{aligned}
P &= (I_B + I_C)V_{CC} \\
&= \frac{1+\beta}{\beta} I_C V_{CC} \\
&= 1 \text{ mW} \\
I_C &= 396\ \mu\text{A} \\
\frac{V_{CC} - V_{BE}}{R_B} &= I_B = \frac{I_C}{\beta} \\
R_B &= \beta \frac{V_{CC} - V_T \ln(I_C/I_S)}{I_C} \\
&= \boxed{453\ \text{k}\Omega} \\
A_v &= -g_m R_C \\
&= -\frac{I_C R_C}{V_T} \\
&= -20 \\
R_C &= \boxed{1.31\ \text{k}\Omega}
\end{aligned}
$$

80)



$A_V = 5$

$R_{out} = R_c = 500\Omega$

$R_E I_c \approx 300\,mV$

$$A_V = \frac{R_c I_c}{R_E I_c + V_T} = \frac{R_c I_c}{300+26} \Rightarrow \begin{array}{l} R_c I_c = 1.63V \Rightarrow I_c = 3.26\,mA \\ R_E I_c \approx 300mV \Rightarrow R_E = 92\Omega \end{array}$$

$$R_1 = \frac{2.5 - (V_{BE} + 0.3)}{10\, I_B}, \quad V_{BE} = V_T \ln\left(\frac{I_c}{I_s}\right) = 0.7624$$

$$10 I_B = 0.326\,mA$$

$$R_1 = \frac{2.5 - (0.7624 + 0.3)}{0.326} = 4.41K$$

$$R_2 = \frac{(0.7624 + 0.3)}{(9)(0.0326)} = 3.62K$$

$V_{CE} = 2.5 - 1.63 - 0.3 = 0.57, \quad V_{BE} = 0.7624.$

Q_1 is in soft saturation region, so active region characteristics still apply.

$R_c = 500\Omega$

$R_1 = 4.41K\Omega$

$R_2 = 3.62K\Omega$ $\Rightarrow$ $A_V = 5$, $R_{out} = 500\Omega$

$R_E = 92\Omega$

5.81

$$R_{out} = R_C \geq 1\text{ k}\Omega$$

To maximize gain, we should maximize R_{out}.

$$R_C = \boxed{1\text{ k}\Omega}$$
$$V_{CC} - I_C R_C - I_E R_E = V_{CE} \geq V_{BE} - 400\text{ mV}$$
$$V_{CC} - I_C R_C - 200\text{ mV} \geq V_T \ln(I_C/I_S) - 400\text{ mV}$$
$$I_C \leq 1.95\text{ mA}$$

To maximize gain, we should maximize I_C.

$$I_C = 1.95\text{ mA}$$
$$I_E R_E = \frac{1+\beta}{\beta} I_C R_E = 200\text{ mV}$$
$$R_E = \boxed{101.5\ \Omega}$$
$$V_{CC} - 10 I_B R_1 - I_E R_E = V_{BE} = V_T \ln(I_C/I_S)$$
$$R_1 = \boxed{7.950\text{ k}\Omega}$$
$$9 I_B R_2 - I_E R_E = V_{BE} = V_T \ln(I_C/I_S)$$
$$R_2 = \boxed{5.405\text{ k}\Omega}$$

5.82

$$
\begin{aligned}
P &= (10I_B + I_C)\,V_{CC} \\
&= \left(10\frac{I_C}{\beta} + I_C\right)V_{CC} \\
&= 5\text{ mW} \\
I_C &= 1.82\text{ mA} \\
I_E R_E &= \frac{1+\beta}{\beta} I_C R_E = 200\text{ mV} \\
R_E &= \boxed{109\ \Omega} \\
A_v &= -\frac{R_C}{\frac{1}{g_m} + R_E} \\
&= -\frac{R_C}{\frac{V_T}{I_C} + R_E} \\
&= -5 \\
R_C &= \boxed{616\ \Omega} \\
V_{CC} - 10I_B R_1 - 200\text{ mV} &= V_{BE} = V_T \ln(I_C/I_S) \\
R_1 &= \boxed{8.54\text{ k}\Omega} \\
9I_B R_2 - 200\text{ mV} &= V_{BE} = V_T \ln(I_C/I_S) \\
R_2 &= \boxed{5.79\text{ k}\Omega}
\end{aligned}
$$

5.83

$$
\begin{aligned}
R_{in} &= \frac{1}{g_m} = 50\ \Omega \text{ (since } R_E \text{ doesn't affect } R_{in}) \\
g_m &= 20\ \text{mS} \\
I_C &= g_m V_T = 520\ \mu\text{A} \\
I_E R_E &= \frac{1+\beta}{\beta} I_C R_E = 260\ \text{mV} \\
R_E &= \boxed{495\ \Omega} \\
A_v &= g_m R_C = 20 \\
R_C &= \boxed{1\ \text{k}\Omega} \\
V_{CC} - 10 I_B R_1 - I_E R_E &= V_{BE} = V_T \ln(I_C / I_S) \\
R_1 &= \boxed{29.33\ \text{k}\Omega} \\
9 I_B R_2 - I_E R_E &= V_{BE} = V_T \ln(I_C / I_S) \\
R_2 &= \boxed{20.83\ \text{k}\Omega}
\end{aligned}
$$

To pick C_B, we must consider its effect on A_v. If we assume the capacitor has an impedance Z_B and $|Z_B| \ll R_1, R_2$, then we have:

$$
A_v = \frac{R_C}{\frac{1}{g_m} + \frac{|Z_B|}{1+\beta}}
$$

Thus, we should choose $\frac{1}{1+\beta} |Z_B| \ll \frac{1}{g_m}$.

$$
\begin{aligned}
\frac{1}{1+\beta} |Z_B| &= \frac{1}{1+\beta} \frac{1}{2\pi f C_B} = \frac{1}{10} \frac{1}{g_m} \\
C_B &= \boxed{1.58\ \mu\text{F}}
\end{aligned}
$$

5.84

$$
\begin{aligned}
R_{out} &= R_C = \boxed{500\ \Omega} \\
A_v &= g_m R_C = 8 \\
g_m &= 16\text{ mS} \\
I_C &= g_m V_T = 416\ \mu\text{A} \\
I_E R_E &= \frac{1+\beta}{\beta} I_C R_E = 260\text{ mV} \\
R_E &= \boxed{619\ \Omega} \\
V_{CC} - 10 I_B R_1 - I_E R_E &= V_{BE} = V_T \ln(I_C/I_S) \\
R_1 &= \boxed{36.806\text{ k}\Omega} \\
9 I_B R_2 - I_E R_E &= V_{BE} = V_T \ln(I_C/I_S) \\
R_2 &= \boxed{25.878\text{ k}\Omega}
\end{aligned}
$$

To pick C_B, we must consider its effect on A_v. If we assume the capacitor has an impedance Z_B and $|Z_B| \ll R_1, R_2$, then we have:

$$
A_v = \frac{R_C}{\frac{1}{g_m} + \frac{|Z_B|}{1+\beta}}
$$

Thus, we should choose $\frac{1}{1+\beta}|Z_B| \ll \frac{1}{g_m}$.

$$
\begin{aligned}
\frac{1}{1+\beta}|Z_B| &= \frac{1}{1+\beta}\frac{1}{2\pi f C_B} = \frac{1}{10}\frac{1}{g_m} \\
C_B &= \boxed{1.26\ \mu\text{F}}
\end{aligned}
$$

5.85

$$
\begin{aligned}
R_{out} &= R_C = \boxed{200\ \Omega} \\
A_v &= g_m R_C = \frac{I_C R_C}{V_T} = 20 \\
I_C &= 2.6\text{ mA} \\
P &= V_{CC}\left(10 I_B + I_C\right) \\
&= V_{CC}\left(10\frac{I_C}{\beta} + I_C\right) \\
&= \boxed{7.15\text{ mW}}
\end{aligned}
$$

5.86

$$
\begin{aligned}
P &= (I_C + 10 I_B) V_{CC} \\
&= \left(I_C + 10 \frac{I_C}{\beta} \right) V_{CC} \\
&= 5 \text{ mW} \\
I_C &= 1.82 \text{ mA} \\
A_v &= g_m R_C \\
&= \frac{I_C R_C}{V_T} \\
&= 10 \\
R_C &= \boxed{143\ \Omega} \\
I_E R_E &= \frac{1+\beta}{\beta} I_C R_E = 260 \text{ mV} \\
R_E &= \boxed{141.6\ \Omega} \\
V_{CC} - 10 I_B R_1 - I_E R_E &= V_{BE} = V_T \ln(I_C/I_S) \\
R_1 &= \boxed{8.210\ \text{k}\Omega} \\
9 I_B R_2 - I_E R_E &= V_{BE} = V_T \ln(I_C/I_S) \\
R_2 &= \boxed{6.155\ \text{k}\Omega}
\end{aligned}
$$

To pick C_B, we must consider its effect on A_v. If we assume the capacitor has an impedance Z_B and $|Z_B| \ll R_1, R_2$, then we have:

$$
A_v = \frac{R_C}{\frac{1}{g_m} + \frac{|Z_B|}{1+\beta}}
$$

Thus, we should choose $\frac{1}{1+\beta} |Z_B| \ll \frac{1}{g_m}$.

$$
\begin{aligned}
\frac{1}{1+\beta} |Z_B| &= \frac{1}{1+\beta} \frac{1}{2\pi f C_B} = \frac{1}{10} \frac{1}{g_m} \\
C_B &= \boxed{5.52\ \mu\text{F}}
\end{aligned}
$$

5.87

$$\begin{aligned}
R_{in} &= \frac{1}{g_m} = 50\ \Omega \text{ (since } R_E \text{ doesn't affect } R_{in}\text{)} \\
g_m &= 20\ \text{mS} \\
I_C &= g_m V_T = 520\ \mu\text{A} \\
A_v &= g_m R_C = 20 \\
R_C &= \boxed{1\ \text{k}\Omega} \\
I_E R_E &= \frac{1+\beta}{\beta} I_C R_E = 260\ \text{mV} \\
R_E &= \boxed{495\ \Omega}
\end{aligned}$$

To minimize the supply voltage, we should allow Q_1 to operate in soft saturation, i.e., $V_{BC} = 400$ mV.

$$\begin{aligned}
V_{BE} &= V_T \ln(I_C/I_S) = 715\ \text{mV} \\
V_{CE} &= V_{BE} - 400\ \text{mV} = 315\ \text{mV} \\
V_{CC} - I_C R_C - I_E R_E &= V_{CE} \\
V_{CC} &= \boxed{1.095\ \text{V}} \\
V_{CC} - 10 I_B R_1 - I_E R_E &= V_{BE} \\
R_1 &= \boxed{2.308\ \text{k}\Omega} \\
9 I_B R_2 - I_E R_E &= V_{BE} \\
R_2 &= \boxed{20.827\ \text{k}\Omega}
\end{aligned}$$

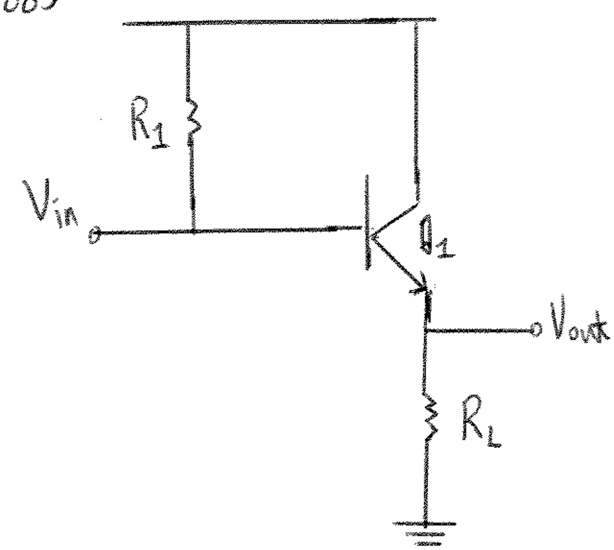
To pick C_B, we must consider its effect on A_v. If we assume the capacitor has an impedance Z_B and $|Z_B| \ll R_1, R_2$, then we have:

$$A_v = \frac{R_C}{\frac{1}{g_m} + \frac{|Z_B|}{1+\beta}}$$

Thus, we should choose $\frac{1}{1+\beta} |Z_B| \ll \frac{1}{g_m}$.

$$\begin{aligned}
\frac{1}{1+\beta} |Z_B| &= \frac{1}{1+\beta} \frac{1}{2\pi f C_B} = \frac{1}{10} \frac{1}{g_m} \\
C_B &= \boxed{1.58\ \mu\text{F}}
\end{aligned}$$

88)



$A_v = 0.85$
$R_{in} > 10k\Omega$
$R_L = 200\Omega$

$$A_v = \frac{R_L}{R_L + \frac{1}{g_m}} = 0.85 \Rightarrow \frac{200}{200 + \frac{1}{g_m}} = 0.85$$

$$\Rightarrow 200 = 0.85\left(200 + \frac{1}{g_m}\right) \Rightarrow \frac{1}{g_m} = 35.294\Omega$$

$$\Rightarrow I_c = \frac{26\,mV}{35.294\Omega} = 0.737\,mA, \quad V_{BE} = V_T \ln\left(\frac{0.737}{6\times10^{-13}}\right) = 0.724V$$

$$R_{in} = R_1 \parallel (r_\pi + (1+\beta)(200\Omega))$$

$$R_{in} = R_1 \parallel 23.73K$$

$$R_{in} = \frac{R_1\, 23.73K}{R_1 + 23.73K} > 10K \Rightarrow R_1 > 17.28K \text{ (Input Impedance requirement)}$$

To support an I_c of 0.737, R_1 must be determined.

88)

$$R_1 = \frac{2.5 - (0.724 + (0.737)(0.2)/0.99)}{0.737/100}$$

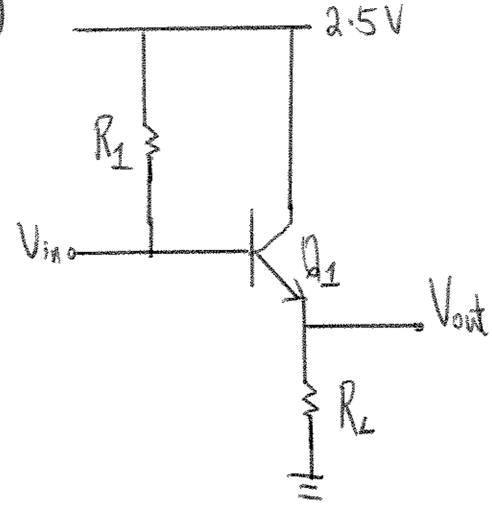
$$R_1 = 220.77K\Omega$$

$$R_1 = 220.77K\Omega \Rightarrow R_{in} = 220.77K\Omega // 23.73K\Omega$$

$$R_{in} = 21.43K\Omega > 10K\Omega$$

$$R_1 = 220.77K\Omega, \quad R_L = 200\Omega \Rightarrow A_v = 0.85, \quad R_{in} = 21.43K\Omega$$

89)



Power = 5 mW

$A_v = 0.9$

$$A_v = \frac{R_L}{R_L + \frac{1}{g_m}} = 0.9 \Rightarrow R_L = 0.9\left(R_L + \frac{1}{g_m}\right)$$

$$R_L = 9\frac{1}{g_m}$$

$$\text{Power} = 2.5\left(I_c + \frac{I_c}{\beta}\right) \Rightarrow I_c = 1.98\,mA$$

$$\frac{1}{g_m} = \frac{V_T}{I_c} = \frac{26mV}{1.98mA} = 13.13\,\Omega$$

$$R_L = (9)(13.13) = 118.17\,\Omega.$$

This is the minimum load resistance, since anything lower will lower the voltage gain.

5.90 As stated in the hint, let's assume that $I_E R_E \gg V_T$. Given this assumption, we can assume that R_E does not affect the gain.

$$
\begin{aligned}
I_E R_E &= 10V_T = 260\text{ mV} \\
A_v &= \frac{R_L}{\frac{1}{g_m} + R_L} = 0.8 \\
g_m &= 80\text{ mS} \\
I_C &= g_m V_T = 2.08\text{ mA} \\
\frac{1+\beta}{\beta} I_C R_E &= 260\text{ mV} \\
R_E &= \boxed{124\ \Omega} \\
V_{CC} - I_B R_1 - I_E R_E &= V_{BE} = V_T \ln(I_C/I_S) \\
R_1 &= \boxed{71.6\text{ k}\Omega}
\end{aligned}
$$

To pick C_1, we must consider its effect on A_v. If we assume the capacitor has an impedance Z_1 and $|Z_1| \ll R_1$, then we have:

$$
A_v = \frac{R_E}{\frac{1}{g_m} + R_E + \frac{|Z_1|}{1+\beta}}
$$

Thus, we should choose $\frac{1}{1+\beta}|Z_1| \ll \frac{1}{g_m}$.

$$
\begin{aligned}
\frac{1}{1+\beta}|Z_1| &= \frac{1}{1+\beta}\frac{1}{2\pi f C_1} = \frac{1}{10}\frac{1}{g_m} \\
C_1 &= \boxed{12.6\text{ pF}}
\end{aligned}
$$

To pick C_2, we must also consider its effect on A_v. Since the capacitor appears in series with R_L, we need to ensure that $|Z_2| \ll R_L$, assuming the capacitor has impedance Z_2.

$$
\begin{aligned}
|Z_2| &= \frac{1}{2\pi f C_2} = \frac{1}{10} R_L \\
C_2 &= \boxed{318\text{ pF}}
\end{aligned}
$$